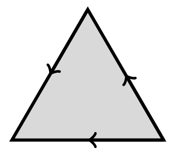
Math 821, Spring 2018
Problem Set #3 (Revised)
Deadline: Friday, March 9, 5:00pm

Instructions: Typeset your solutions in LaTeX. You are encouraged to use the Math 821 header file. Email your solutions to Jeremy (jlmartin@ku.edu) as a PDF file named with your last name and the problem set number (e.g., `Thurston3.pdf`). Collaboration is encouraged, but each student must write up his or her solutions independently and acknowledge all collaborators.

Problem #1 The *dunce hat* is the space D obtained from a triangle by identifying all three edges with each other, with the orientations indicated below. Prove that D is simply-connected (i) using Van Kampen's theorem; (ii) using what you know about 2-dimensional cell complexes.



Problem #2 (Hatcher, p.53, #4, modified) Let $n \geq 1$ be an integer, and let $X \subset \mathbb{R}^3$ be the union of n distinct rays emanating from the origin. Compute $\pi_1(\mathbb{R}^3 \setminus X)$.

Problem #3 Let $a_1, \ldots, a_n$ be nonzero integers. Construct a cell complex X from S^1 as follows: For each $j = 1, \ldots, n$, attach a 2-cell to S^1 by wrapping it around the circle a_j times. Compute $\pi_1(X)$.

Problem #4 (Hatcher, p.53, #6, modified) Let X be a path-connected cell complex, and let Y be a cell complex obtained from X by attaching an n-cell for some $n \geq 3$. Show that the inclusion $X \hookrightarrow Y$ induces an isomorphism $\pi_1(X) \cong \pi_1(Y)$.

Problem #5 (Hatcher p.79, #9) Show that if a path-connected, locally path-connected space X has finite fundamental group, then every map $X \to S^1$ is nullhomotopic. (Hint: Use the covering space map $\mathbb{R} \to S^1$.)

Problem #6 (Hatcher p.80, #12) Let a and b be the generators of $\pi_1(S^1 \vee S^1, x_0)$ corresponding to the two copies of S^1, with x_0 their common point. Draw a picture of the covering space $\tilde{X}$ of $S^1 \vee S^1$ corresponding to the normal subgroup of $\pi_1(S^1 \vee S^1)$ generated by a^2, b^2, and $(ab)^4$, and prove that this covering space is indeed the correct one. (I.e., this group should be $p_*\pi_1(\tilde{X}, \tilde{x}_0)$.)

Problem #7 (Hatcher p.80, #18) For a path-connected, locally path-connected, and semilocally simply-connected space X, call a path-connected covering space $\tilde{X} \to X$ *abelian* if it is normal and has abelian deck transformation group. Show that X has an abelian covering space $\hat{X}$ that is a covering space of every other abelian covering space of X, and that such a "universal abelian covering space" is unique up to covering space isomorphism. Describe this covering space explicitly for $X = S^1 \vee S^1$ and $X = \bigvee^3 S^1$ (i.e., three circles, with a point from each identified).