Math 821, Spring 2014
Problem Set #5
Due date: Friday, April 4

Problem #1 Verify that the simplicial boundary map defined by

$$\partial_n[v_0,\dots,v_n] = \sum_{i=0}^{n}(-1)^i[v_0,\dots,\widehat{v_i},\dots,v_n]$$

satisfies the equation $\partial_{n-1}\circ\partial_n = 0$ for all n. (Yes, this calculation is done explicitly in Hatcher. But it is so important that everyone should do it for themselves at least once.)

Problem #2 Let X be an abstract simplicial complex on vertex set $[n]$ and let $|X|$ be a geometric realization of X (not necessarily the standard one — it doesn't matter). What invariant of $|X|$ corresponds to the dimension of $H_0^\Delta(X)$?

Problem #3 Consider the matrix

$$M = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

Describe $\operatorname{coker} M$ (i) if M is regarded as a linear transformation over $\mathbb{Q}$; (ii) if M is regarded as a linear transformation over $\mathbb{Z}$; (iii) if M is regarded as a linear transformation over $\mathbb{F}_q$ (the finite field with q elements).

Problem #4 [Hatcher p.131 #4] Compute <u>by hand</u> the simplicial homology groups of the "triangular parachute" obtained from Δ^2 by identifying its vertices to a single point.

Problem #5 [Hatcher p.131 #5] Compute <u>by hand</u> the simplicial homology groups of the Klein bottle using the Δ-complex structure on p.102 (with two triangles).

Problem #6 Check your answers on the last two problems using `Macaulay2` or your favorite computer algebra system. Submit your source code and output with your problem set. (For example, if you are using `Macaulay2`, you can cut and paste the session transcript into your TeX file and use the `verbatim` enviroment.)

Here is a link to how to get started with Macaulay2.

Problem #7 Let $\Delta^{n,d}$ denote the d-skeleton of the n-simplex. As an abstract simplicial complex, Δ is generated by all $(d+1)$-element subsets of $\{0,\dots,n\}$. Use `Macaulay2` (or another computer algebra system) to compute the homology groups of $\Delta^{n,d}$ for various values of n and d. Conjecture a general formula for $H_k(\Delta^{n,d})$ in terms of n, d and k. (Prove it, if you want.)