Instructions: Do all problems and typeset them in LaTeX. E-mail the PDF file to Jeremy at jlmartin@ku.edu under the filename `your-last-name.pdf` by **Friday, April 1, 5:00pm.** You are encouraged to use the LaTeX header file and to refer to Jeremy's lecture notes.

Problem #1 Complete the proof that $a(G) = |p_G(-1)|$ for all graphs G, where $a(G)$ is the number of acyclic orientations and p_G is the chromatic polynomial. (The proof is sketched in the lecture notes. Your main task is to prove the deletion/contraction recurrence for $a(G)$.)

Problem #2 Let $\mathcal{I} = \{I_1, \dots, I_n\}$, where each I_i is an interval $[a_i, b_i] \subseteq \mathbb{R}$. The corresponding **interval graph** is defined as the graph $G_\mathcal{I}$ with vertices $1, 2, \dots, n$ and edges ij whenever $I_i \cap I_j \neq \varnothing$. Prove that $G_\mathcal{I}$ is always a chordal graph.

Problem #3 Find and prove a formula for $p_{C_n}(k)$ that holds for all $n \geqslant 3$. You may want to start by gathering data. To do this using Sage, go to http://aleph.sagemath.org and type this into the box:

```
G = graphs.CycleGraph(3)
G.chromatic_polynomial()
```

You can change the value of 3 to gather data on other cycle graphs (to do this most efficiently, use a `for` loop). If you want to see the polynomial in factored form, change the second line to

```
G.chromatic_polynomial().factor()
```

In either case, the coefficients you see should look familiar, enabling you to make a conjecture about what $p_{C_n}(k)$ is for general n. By the way, does the formula work for $n < 3$?

Problem #4 A **regular polyhedron** (or **Platonic solid**) is a polyhedron in which all vertices have the same number of neighbors, all edges have the same length, all edge-edge angles are equal, and all face-face angles are equal—in other words, as symmetric as it can possibly be. The graph G of any regular polyhedron is planar (embed G on the surface of a sphere, then puncture the sphere to obtain a plane drawing). Moreover, G is k-regular for some $k \geqslant 3$, and G^* is ℓ-regular for some $\ell \geqslant 3$.

Using handshaking, the length-sum formula, and Euler's formula, determine all possibilities for k and ℓ, and thus for the numbers of vertices, edges and faces of P. For at least three of these possibilities (preferably all of them), show that there is only one planar graph (up to isomorphism) with these parameters, and determine what graph it is.

Problem #5 A graph is **outerplanar** if it has a drawing in which every vertex lies on the unbounded face.

(#5a) Show that K_4 and $K_{3,2}$ are planar but not outerplanar. (Hint: Mimic the proof that K_5 and $K_{3,3}$ are not planar.)

(#5b) Use Kuratowski's theorem to prove that G is outerplanar if and only if it has neither K_4 or $K_{3,2}$ as a minor. (Hint: One direction follows from part (a). For the converse, find an appropriate modification of G to which you can apply Kuratowski's theorem.