Instructions: Do all problems and typeset them in LaTeX. E-mail the PDF file to Jeremy at `jlmartin@ku.edu` under the filename `your-last-name.pdf` by **Friday, March 11, 5:00pm.** You are encouraged to use the LaTeX header file and to refer to Jeremy's lecture notes.

Problem #1 Let G be a k-regular graph of even order that is $(k-1)$-edge-connected (i.e., $G-A$ is connected for any edge set A of size $k-2$ or less). Prove that G has a perfect matching. (Hint: Adapt the proof of Petersen's theorem, Thm. 3.26 in the lecture notes.)

Problem #2 Let G be a simple graph.
(#2a) Let $C, C' \subseteq G$ be cycles. Prove that $C \triangle C'$ is the (edge-)disjoint union of cycles.
(#2b) Let $B, B' \subseteq G$ be cuts. Prove that $B \triangle B'$ is the (edge-)disjoint union of cuts.

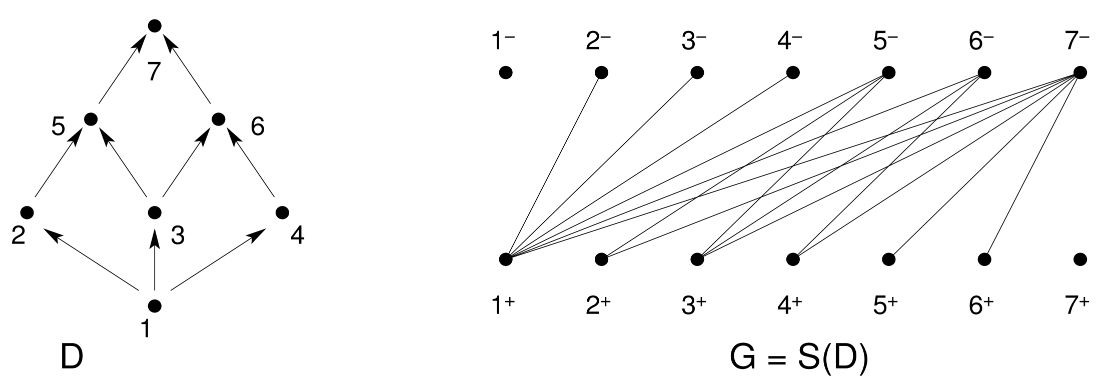
Problem #3 Let G be a connected simple graph, and let $F \subseteq E(G)$ be a nonempty set of edges. Prove that F is an edge cut if and only if $|F \cap C|$ is even for every cycle $C \subseteq G$.

Problem #4 Derive the König-Egerváry Theorem from the Max-Flow/Min-Cut Theorem. (Hint: Given an arbitrary X,Y-bipartite graph G, transform it into a source-sink network N in such a way that flows and cuts in N correspond respectively to matchings and vertex covers in G.)

Problem #5 Prove Dilworth's Theorem by the following steps:

(#5a) Prove that if $\mathcal{P} = \{P_1, \dots, P_n\}$ is any path cover of a digraph D and $I = \{v_1, \dots, v_m\}$ is an independent set in D, then $n \geqslant m$. (This is weak duality, and it is the easier part.)

(#5b) Given a digraph D, define its *split* as the undirected, bipartite graph $G = S(D)$ with two vertices v^+, v^- for each vertex $v \in V(D)$, and an edge u^+v^- for each edge $\overrightarrow{uv}$ in D. (This is a similar construction to the one used to derive vertex-Menger from MFMC.)



Now apply the König-Egerváry Theorem to $S(D)$ and translate the result into Dilworth's Theorem for D.