Instructions: Do all problems and typeset them in LaTeX. E-mail your final PDF file to Jeremy at jlmartin@ku.edu by **Friday, January 29, 5:00pm.** You are encouraged to use the header file at

http://www.math.ku.edu/~jmartin/math725/header.tex.

Problem #1 Prove Theorem 1.5.1 in Diestel (equivalence of the various characterizations of trees). You may use anything in Diestel that occurs before that theorem.

Problem #2 The **complement** of a simple graph $G = (V, E)$ is the simple graph $\overline{G}$ on the same vertex set, in which two vertices are adjacent in $\overline{G}$ if and only if they are *not* adjacent in G.

(#2a) Describe $\overline{C_3}$, $\overline{C_4}$, $\overline{C_5}$ and $\overline{K_{m,n}}$.
(#2b) What is the smallest graph (other than K_1) that is isomorphic to its complement?
(#2c) Prove that for every simple graph G, at least one of G or $\overline{G}$ is connected.

Problem #3 Let Q_n be the n-dimensional cube. (Recall that $V(Q_n)$ can be regarded as the set of bit strings of length n, with two bit strings adjacent iff they differ in exactly one bit.) For $0 \leqslant k \leqslant n$, how many different isomorphic copies of Q_k are there in Q_n? Give your answer as a general formula. It may help to consider extreme cases first (e.g., $k = 0, 1, n-1, n$).

Problem #4 Let R_n be the graph on the bit strings of length n, in which two bit strings are adjacent if and only if they *agree* in exactly one bit. Show that $R_n \cong Q_n$ if and only if n is even. (For odd n, find some property that Q_n has and R_n lacks; it will help to draw R_3 explicitly and stare at the drawing for a few minutes. For even n, find an explicit isomorphism.)

Problem #5 The **odd graph** O_n is defined as follows. The vertices are the subsets of $[2n+1]$ of size n, with two vertices adjacent if and only if they are disjoint as sets. How many edges does O_n have?

Problem #6 Count the number of spanning trees of K_n for all $n \leqslant 5$ by brute force. (Hint: First figure out all the possible isomorphism classes of trees of each order n, then calculate how many copies of each tree occur as subgraphs of K_n.)

Problem #7 Prove that every set of six people contains either a set of three mutual acquaintances or a set of three mutual strangers. (Begin by reformulating the problem in graph-theoretic language.)