Math 724, Fall 2017
Homework #1
Deadline: Friday, September 1, 5:00pm

Instructions: Typeset your solutions in LaTeX. Email your solutions to Jeremy (jlmartin@ku.edu) as a PDF file named with your last name and the problem set number (e.g., `Abel1.pdf`). Collaboration is encouraged, but each student must write up his or her solutions independently and acknowledge all collaborators.

(#1) Problem #13.

(#2) Chapter 1 Supplementary Problem #1.

(#3) Chapter 1 Supplementary Problem #2.

(#4) Chapter 1 Supplementary Problem #8. Give at least two proofs, including at least one that is purely combinatorial (i.e., interpret both sides of the equation as different ways of counting the same set).

(#5) A poker hand consists of five cards, drawn from a standard 52-card deck.

(6a) How many different possible poker hands are there?

(6b) A *flush* is a hand with five cards of the same suit (e.g., ♠A8732 or ♣KJ984). How many different flushes are there?

(6c) A *full house* is a hand with three cards of one rank and two of another (e.g., ♠A ♡8 ♢8 ♣A8 or ♠63 ♡63 ♣6). How many different full houses are there?

(#6) Let $a_n = \sum_{k=0}^{n} \binom{n}{k}^2$. Calculate a_n for $0 \leq n \leq 3$. Then stare at Pascal's triangle and make a conjecture about the value of a_n. If you like, use Sage or another computer algebra system to check that your conjecture works for a few more values of n. Prove your conjecture *combinatorially*; that is, find a set that can be counted in two ways so that one way of counting gives the formula for a_n, and the other way gives the formula you have conjectured.