

Math 243, Fall 2015
Extra Honors Problem for Week 4

Consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by

$$f(x, y) = \begin{cases} (x^2 + y^2) \sin((x^2 + y^2)^{-1/2}) & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

Show that f is continuous and differentiable at $(0, 0)$, but is not of class C^1 because its partial derivatives are not continuous there. You should definitely find out what the graph looks like, because it's pretty wacky. You can use [this Java applet](#) (which has a hint on the problem that may or may not help you), or you can draw your own graph using Sage, Matlab, Grapher, etc. Be sure to zoom in a few times to see what the graph looks like.