

Math 141 Honors Problems #13

Due date: Tuesday, 11/24/09

**HP21 [3 points]** Find a formula for

$$\int e^{ax} \sin bx \, dx$$

in terms of  $a$  and  $b$  (where  $a$  and  $b$  are real numbers).

**HP22 [3 points]** Let  $p(x)$  be a polynomial of degree  $n$ , say

$$p(x) = \sum_{k=0}^n a_k x^k$$

where  $a_0, a_1, \dots, a_k, \dots, a_n$  are real numbers. Find a formula for

$$\int e^x p(x) \, dx$$

in terms of the  $a_k$ 's.

**HP23 [4 points]** As discussed in class, there's no closed formula for

$$\int \frac{e^x}{x} \, dx.$$

For similar reasons, there is no closed formula for

$$\int \frac{e^x}{x^2} \, dx.$$

On the other hand, there *are* similar-looking functions which can be antiderivated. For instance, the Quotient Rule tells us that  $\frac{d}{dx}(e^x/x) = (xe^x - e^x)/x^2 = e^x/x - e^x/x^2$ , which implies that

$$\int \left( \frac{e^x}{x} - \frac{e^x}{x^2} \right) dx = \frac{e^x}{x} + C.$$

(23a) For which constants  $a, b, c$  can the integral

$$\int \left( \frac{ae^x}{x} + \frac{be^x}{x^2} + \frac{ce^x}{x^3} \right) dx$$

be evaluated?

(23b) Can you say anything more generally about integrals of the form

$$\int \left( \sum_{k=1}^n \frac{a_k e^x}{x^k} \right) dx?$$