Math 141 Homework #7
Due Tuesday, 10/2/07
Extra Problem

Problem #1 If n is a positive integer, define a polynomial function $f_n(x)$ by

$$f_n(x) = \sum_{i=0}^{n} \frac{x^i}{i!}$$

(#1a) Write down explicit expressions for $f_n(x)$ for a few small values of n (say $0 \leq n \leq 5$). (To get you started, $f_0(x) = \sum_{i=0}^{0} \frac{x^i}{i!} = x^0/0! = 1$.)

(#1b) Calculate the derivatives $f_n'(x)$ of the functions you wrote down in part (a). What do you notice?

(#1c) Define a new function, with the curious-looking name $f_\infty(x)$, by

$$f_\infty(x) = \lim_{n \to \infty} f_n(x)$$

(you may have to think a bit about how to make this definition make sense). Based on your solution to part (b), what would you expect about $f_\infty'(x)$?

(#1d) By evaluating $f_n(1)$ for a few values of n, make a conjecture about the value of $f_\infty(1)$.

(#1e) Based on your answers to parts (d) and (e), what function does $f_\infty(x)$ remind you of? Evaluate that function and $f_\infty(x)$ at a few other values of x to see what else the two functions have in common.