Math 141 Homework #5
Due Tuesday, 9/18/07
Extra Problems

Problem #1 We know that every polynomial function $f(x)$ can be written in the form

$$f(x) = \sum_{i=0}^{n} a_i x^i$$

where $n \geq 0$ is an integer and $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers.

Write down a formula for $f'(x)$ in terms of the numbers $a_n, a_{n-1}, \ldots, a_1, a_0$ and the variable x. You should express your answer using summation notation, but you don't have to prove it.

Problem #2 Let $f(x)$ and $g(x)$ be functions, and let a and b be constants. Using the definition of the derivative of a function, write down a clear, well-organized proof of the fact that

$$\frac{d}{dx}\left(a \cdot f(x) + b \cdot g(x)\right) = a \cdot f'(x) + b \cdot g'(x).$$

You are encouraged to use the proofs of the Constant Multiple, Sum, and Difference Rules (textbook pp. 186–187, or from class on Wednesday 9/12) as templates in constructing your own proof.

Problem #3 Construct a function $q(x)$ with domain $\mathbb{R}$ that is differentiable but not second-differentiable. That is, $q'(x)$ is defined and continuous on $\mathbb{R}$, but $q''(x)$ has a discontinuity (say, at $x = 0$).

Bonus part: For all positive integers n, construct a function q such that q is $(n-1)^{th}$-order differentiable but not n^{th}-order differentiable. That is,

$$q, \quad \frac{dq}{dx}, \quad \frac{d^2q}{dx^2}, \quad \ldots, \quad \frac{d^{n-1}q}{dx^{n-1}}$$

are all defined and continuous, but $\dfrac{d^nq}{dx^n}$ has a discontinuity (again, say, at $x = 0$).