Math 141 Homework #1
Due Tuesday, 8/21/07

This homework is an exception to the general grading policy announced in the syllabus (about required problems and practice problems). I'm not necessarily interested in correct solutions, but I do want you to make a good-faith effort to solve every problem on your own. If I can see that you've done so, you'll earn full credit.

#1. As discussed in class on Thursday (and as you may have already known), if you add up the binomial coefficients in the n^{th} row of Pascal's Triangle, you get 2^n. That is,

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n.$$

What happens if you *square* numbers before adding them? That is, if you define a function $f : \mathbb{N} \to \mathbb{N}$ by the formula

$$f(n) = \sum_{k=0}^{n} \binom{n}{k}^2$$

then is there a simpler formula for $f(n)$? Calculate the value of $f(n)$ for a few small values of n, then try and make a conjecture. (Hint: Look at Pascal's Triangle again.) *Bonus problem* (hard): Once you've found the answer, can you explain why it is always true?

Note: I know that you can find the formula for $f(n)$ from external sources; for example, it's right there on the Wikipedia page on Pascal's Triangle. Please don't. The point of this problem is for you to find it yourself (which ought to be a lot more fun than just looking it up). I'm not going to base any part of your grade on just writing down the correct formula.

#2. Factorials are hard to calculate: to find the value of $n!$, you need to perform n separate multplications, involving larger and larger numbers. Even using the factorial button on a calculator, this can take a while if n is large (even assuming your calculator can handle numbers as big as $n!$ in the first place; my TI-83 Plus can only compute $n!$ when $n \leq 69$). Fortunately, there is an amazingly good way to *approximate* factorials, called *Stirling's formula*:

$$s(n) = n^n e^{-n} \sqrt{2\pi n}.$$

(a) Calculate the numbers $1!$, $2!$, ..., $15!$. Then calculate $s(1)$, $s(2)$, ..., $s(15)$. (You can avoid a certain amount of tedium by using your calculator to define $s(n)$ as a function and making a table of values.)

(b) One way to measure the error of this approximation is to calculate the numbers

$$n! - s(n)$$

for all values of n. (The closer this is to 0, the better the approximation.) Another possibility is to calculate the numbers

$$\frac{n!}{s(n)}$$

for all values of n. (The closer this is to 1, the better the approximation.)

Try calculating both $n! - s(n)$ and $n!/s(n)$ for several values of n (not necessarily all of them, but enough so that you can see a pattern). What is happening as n gets larger and larger? Which of these two methods do you think is a better measurement of the accuracy of the approximation? (There's not necessarily a right answer to this question.)

#3. For this problem, in contrast, I do want you to use an external source on a real-life function of great interest in Kansas this week: the heat index. Start by reading the Wikipedia article about the heat index.

(a) Suppose that you regard the heat index as a function of both temperature and humidity. What are the domain and range? Is the function one-to-one?

(b) How can you regard the heat index as a function of temperature alone? (Of course, you're going to have to figure out what role humidity is going to play in this new function.) What are the domain and range of this function? Is it one-to-one? What if you regard heat index as a function of humidity alone?

(c) The Wikipedia page contains two very different-looking formulas for the heat index. Compare the two functions. Do they produce the same values? (Note: One formula measures temperature in degrees Fahrenheit and the other uses degrees Celsius, so you will have to change units before comparing them.) If they don't agree, what do you think is going on?

Bonus problem: A Wikipedia editor named `cmh` made the following comment on the article's talk page:

> This clause is confusing: "thus there is an inverse relationship between maximum potential temperature and maximum potential relative humidity, therefore making, say, a simultaneous temperature of 120°F (50°C) and 90% relative humidity physically impossible." There cannot be an inverse relationship between two maximums.

Is this criticism correct?