MATH 121 - Fall 2007

Additional Review Problems for the Final Examination - Covers [4.2, 6.7] in Stewart

Note: These problems are in addition to the Review Problems for the Midterm Examination (which covers [1.1, 4.1] in Stewart). A portion of the Final Examination will include material from [1.1, 4.1] in Stewart.

1. Find the interval(s) on which the function is increasing. Find the interval(s) on which the function concave upward.

(a) $f(x) = 2x^3 - 3x^2 - 12x + \pi^2$ (b) $g(x) = x^3 - 3x^2 + 3x + 17$

(c) $h(t) = \frac{t^4}{16} - \frac{t^3}{3} + \frac{3t^2}{8}$ (d) $s(u) = u^3 - 12u^2 + 2$

(e) $g(t) = \frac{\ln t}{t}$ (f) $h(u) = 2ue^u$

2. Let $f(x) = 2x - \frac{2}{x} - 4$. Find the domain of f. Determine the zeros of f. Find the intervals on which f is increasing. Find the intervals on which f is concave downward.

3. Let g be the function defined by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$. For $0 \leq t \leq 8$, the function g is decreasing most rapidly when $t =$ [Hint: Use a calculator.]

(A) 0.949 (B) 2.017 (C) 3.106 (D) 5.965 (E) 8.000

4. Given $f'(x) = \cos(2x) - \sin x$, $0 < x < 2\pi$. On which open intervals is the function f increasing?

5. Find the absolute maximum and minimum values of $f(x)$ and the corresponding x-values on the given interval.

(a) $f(x) = x^3 - 3x^2 + 2$, $[-2, 3]$ (b) $f(x) = \frac{2x}{x^2+4}$, $[0, 3]$

(c) $f(x) = x - 2\sin x$, $[0, \pi]$ (d) $f(x) = xe^{-x^2}$, $(-\infty, \infty)$

6. The absolute maximum and minimum values of $y = x^3 - 9x + 8$ on the interval $[-3, 1]$ are

(A) $8 + 6\sqrt{3}$, 8 (B) $8 + 6\sqrt{3}$, 0 (C) 8, 0 (D) $8 - 6\sqrt{3}$, 0 (E) None of these.

7. At what value of x does the function $f(x) = 3x - x^{1/3}$ change from increasing to decreasing?

8. On what interval(s) is the function $g(t) = \frac{t}{t^2+1}$ decreasing?

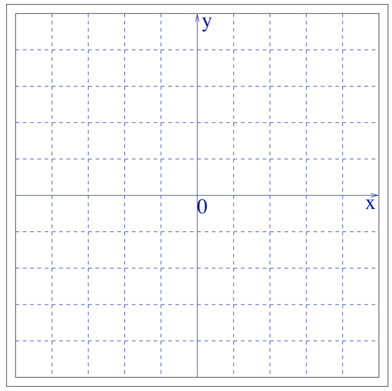
9. How many points of inflection does $h(x) = x^3e^{-x}$ have?

10. Let $f(x) = x^4 - 4x^2$.

(a) Find the critical numbers of f, the intervals on which f is increasing or decreasing. Find the (x, y) coordinates of any local extrema.

(b) Find the inflection points of f and the intervals on which f is concave upward or concave downward.

(c) Sketch the graph of f by using the information obtained in (a) and (b).

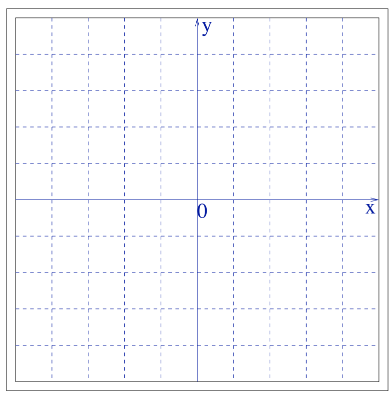


11. If $f(x) = x^{\frac{1}{3}}(4-x)^{\frac{2}{3}}$, then $f'(x) = \dfrac{4-3x}{3x^{\frac{2}{3}}(4-x)^{\frac{1}{3}}}$.

(a) Find the critical numbers of the function f. What is the domain of f'?

(b) Determine the intervals on which f is increasing or decreasing.

(c) Sketch a rough graph of f below.



12. Evaluate the limit.

(a) $\displaystyle\lim_{x\to\infty} \frac{\ln\left(x^3\right)}{x^2}$ (b) $\displaystyle\lim_{x\to\infty} \frac{e^x - x}{e^x + x}$ (c) $\displaystyle\lim_{x\to\infty} \frac{2 - x + x^2}{2 + x - 6x^2}$

(d) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x} e^x$ (e) $\displaystyle\lim_{x\to\infty} e^{-x}\sin x$ (f) $\displaystyle\lim_{x\to\sqrt{2}} \left(\frac{x^2 - 2}{x - \sqrt{2}}\right)$

(g) $\displaystyle\lim_{x\to 1^-} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$ (h) $\displaystyle\lim_{x\to \pi/2} \left(\frac{\pi}{2} - x\right)\tan x$ (i) $\displaystyle\lim_{x\to 1} x^{\frac{1}{1-x}}$

13. If $a, b > 0$, find the value of $\displaystyle\lim_{x\to 0} \frac{a^x - b^x}{x}$.

14. A three-sided fence is to be built next to a straight section of river, which forms the fourth side of a rectangular region. The enclosed area is to equal 1800 ft^2. Find the dimensions of the enclosure to minimize the fence material.

15. A three-sided fence is to be built next to a straight section of river, which forms the fourth side of a rectangular region. There are 96 ft of fencing available. Find the maximum enclosed area and the dimensions of the corresponding enclosure.

16. A mathematician wishes to mail popcorn in a cylindrical package of length h and a circular base of radius r. Because of the post office regulations, the length of the cylinder plus the circumference of the base cannot be more than 108 inches.

(a) Express the volume V of the package in terms of the radius r.

(b) Find the dimensions of the cylindrical package with maximum volume. Justify your answer.

17. What is the maximum possible area of a rectangle with a base that lies on the x axis and with two upper vertices lying on the graph of $y = 9 - x^2$. Justify your answer.

18. Suppose that a rectangular box with open top and square base is to be made using two different materials. The material for the base cost \$2 per square foot and the material for the four sides costs \$1 per square foot. Find the dimensions of the box of greatest volume subject to the condition that \$96 is spent for the material. What is the maximum volume? Justify your answer.

19. Find the points on the hyperbola $x^2 - y^2 = 16$ closest to the point $(0, 2)$.

20. Amy is setting up a lemonade stand. The cost for making x glasses of lemonade is $5 + 0.02x$ dollars. Previous experience indicates that she can sell 80 glasses of lemonade at a price of \$0.50 per glass and that for each \$0.10 increase in price, she will sell 4 fewer glasses. At what price should the lemonade be sold to maximize the profit?

21. The velocity of a wave of length L in deep water is

$$v = K\sqrt{\frac{L}{C} + \frac{C}{L}}$$

where K and C are known positive constants. What is the length of the wave that gives the minimum velocity?

22. Given the cost function (in dollars)

$$C(x) = 2\sqrt{x} + \frac{x^2}{8000},$$

find

(a) the average cost function,

(b) the marginal cost function,

(c) the production level that minimizes the average cost,

(d) the minimum average cost.

23. Given the cost function and the demand function

$$C(x) = 16,000 + 500x - 1.6x^2 + 0.004x^3, \qquad p(x) = 1700 - 7x,$$

find

(a) the production level that maximize the revenue and the maximum revenue,

(b) the production level that maximize the profit and the maximum profit,

(c) the consumer surplus when the sale level is $x = 100$.

24. The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is \$800 per month. A market survey suggests that, on the average, one additional unit will remain vacant for each \$10 increase in rent. What rent should the manager charge to maximize revenue?

25. (a) Give the iterative formula for Newton's method for approximating a root of the equation $f(x) = 0$.

(b) Use Newton's method with initial approximation $x_1 = 2$ to estimate the solution of the equation $x^3 - 2x - 8 = 0$ in the interval (2, 3) accurate to eight decimal places. Give your sequence of approximations.

26. A major league pitcher can throw a baseball with an initial velocity of 144 ft/sec. If he throws the ball straight up, how high will it go? (Neglect air resistance and use $g = -32$ ft/sec^2.)

27. A particle moves along the y-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = t\cos t$. At time $t = 0$, the position of the particle is $y = 3$.

(a) For what intervals of t, $0 \leq t \leq 5$, is the particle moving upward?

(b) Write an expression for the acceleration $a(t)$ of the particle in terms of t.

(c) Write an expression for the position $y(t)$ of the particle in terms of t.

(d) For $t > 0$, find the position of the particle the first time when the velocity of the particle is zero.

28. A particle with velocity at any time t given by $v(t) = e^t$ moves along a straight line. How far does the particle move from time $t = 0$ to $t = 2$?

29. (a) State the trapezoidal approximation T_6 for an arbitrary function f on the interval $[a, b]$.

(b) In a three hour trip, the velocity of a car at each half hours was recorded as follows:

Time(Hours)	0	.5	1	1.5	2	2.5	3
Velocity(MPH)	0	40	55	50	35	30	0

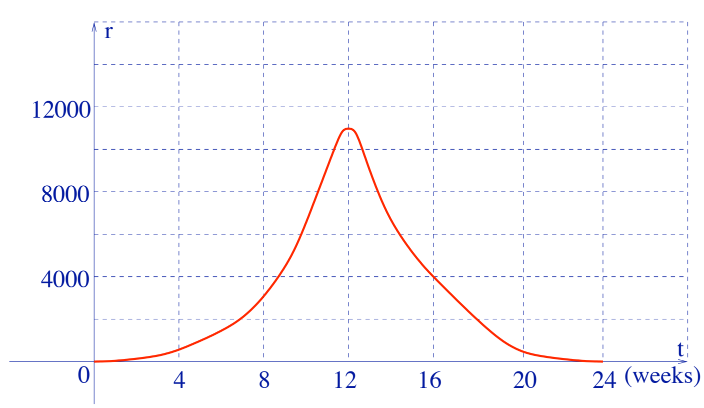
Estimate the distance traveled by using the trapezoidal approximation T_6.

(c) Estimate the average velocity of the car during the trip.

30. Calculate (a) the trapezoidal approximation T_6, (b) the midpoint approximation M_6, (c) the Simpson's approximation to $\int_0^3 \sqrt{x^2+1}\,dx$.

31. Let R be the region enclosed by the curve $y = \ln x$, the x axis, and the lines $x = 1$ and $x = 5$. Use the trapezoidal rule with $n = 4$ to approximate R.

32. A population of honeybees increased at a rate of $r(t)$ bees per week, where the graph of r is as shown. Use the (a) trapezoidal rule, (b) Simpson's rule, with six subintervals to estimate the increase in the bee population during the first 24 weeks.



33. If $2 \le f'(t) \le 3$ for all t in $[1, 5]$, then

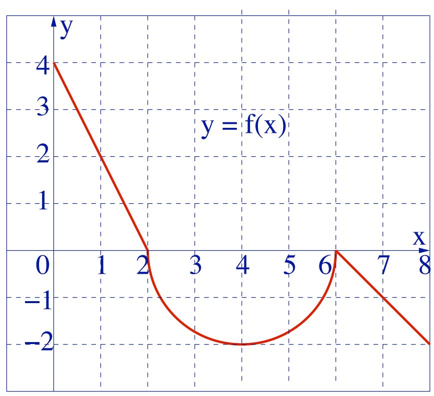
(a) $f(t) > 0$ on $[1, 5]$
(b) $8 \le f(5) \le 12$
(c) $8 \le f(5) - f(1) \le 12$
(d) $f(t)$ is concave upward on $[1, 5]$
(e) $f(t)$ is decreasing on $[1, 5]$

34. If $\int_0^3 f(x)\,dx = 5$ and $\int_2^3 f(x)\,dx = 3$, then $\int_2^0 (2x - 3f(x))\,dx =$

(a) 2 (b) -10 (c) 10 (d) -2 (e) 6

35. The graph of $f(x)$ consists of two straight lines and a semicircle. Use it to evaluate each integral.

(a) $\int_0^2 f(x)\,dx$ (b) $\int_2^4 f(x)\,dx$ (c) $\int_4^7 f(x)\,dx$ (d) $\int_0^8 f(x)\,dx$.



36. Find $f(x)$.

(a) $f'(x) = 5x^4 - 2x^5, \quad f(1) = 2$
(b) $f'(x) = 1 + 2\sin x - \cos x, \quad f(0) = 3$
(c) $f''(x) = 3e^x + 5\sin x, \quad f(0) = 1, \quad f'(0) = 2$
(d) $f''(x) = x^3 + x, \quad f(0) = 1, \quad f(1) = 2$

37. Find the derivative of the function.

(a) $f(x) = \int_0^x t^2 \sin t\,dt$ (b) $h(u) = \int_u^1 \sqrt{1 + x^4}\,dx$
(c) $g(x) = \int_1^{x^2} \cos(t^2)\,dt$ (d) $k(t) = \int_{\sqrt{t}}^t \frac{e^x}{x}\,dx$

38. Evaluate the integral.

(a) $\int e^{\sin x} \cos x\,dx$ (b) $\int \frac{x+1}{x^2 + 2x + 5}\,dx$ (c) $\int_0^2 |\sin(\pi x)|\,dx$
(d) $\int \frac{1}{(1 - 4x)^2}\,dx$ (e) $\int x(\sin x)\,dx$ (f) $\int_0^2 6x(x^2 + 2)^2\,dx$
(g) $\int_1^3 2e^{-3x}\,dx$ (h) $\int x^2 \sqrt{1 + x^3}\,dx$ (i) $\int_0^{\pi/2} (\cos x)\sqrt{\sin x}\,dx$
(j) $\int_4^6 x\sqrt{x - 4}\,dx$ (k) $\int_2^3 \left(x + \frac{1}{x}\right)^2 dx$ (l) $\int (xe^x + e^{1+x})\,dx$

(m) $\displaystyle\int \frac{dt}{9t^2+4}$ (n) $\displaystyle\int x(\ln|x|)\,dx$ (o) $\displaystyle\int_0^{\pi/3} \sin(3x)\,dx$

(p) $\displaystyle\int_1^2 \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right) dx$ (q) $\displaystyle\int_0^1 xe^{x^2}\,dx$ (r) $\displaystyle\int_1^e (\ln x)\,dx$

(s) $\displaystyle\int \frac{dx}{\sqrt{x}(1+\sqrt{x})^2}$ (t) $\displaystyle\int x^2\sec^2(x^3-\pi)\,dx$ (u) $\displaystyle\int_{\pi/2}^{\pi} \frac{4\cos u}{(1+\sin u)^2}\,du$

(v) $\displaystyle\int_1^e \frac{(2+\ln x)^2}{x}\,dx$ (w) $\displaystyle\int e^{2x}\cos x\,dx$ (x) $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

39. Let $F(x)=\displaystyle\int_0^x \sqrt{\tan t}\,dt$. What is the value of $F'(0.05)$?

(a) 0.05 (b) 0.2237 (c) 2.2351 (d) 2.241 (e) 0.0075

40. $\displaystyle\frac{d}{dx}\int_0^{x^2} \cos^2 t\,dt =$

(a) $\cos^2 x$ (b) $\cos^2 x^2$ (c) $\sin^2 x^2$ (d) $2x\cos^2 x^2$ (e) $x^2\cos^2 x^2$

41. For all real b, $\displaystyle\int_0^b |2x|\,dx$ is

(a) $-b|b|$ (b) b^2 (c) $-b^2$ (d) $b|b|$ (e) none of these

42. If the function g has a continuous derivative on $[0, c]$, then $\displaystyle\int_0^c g'(x)\,dx =$

(a) $g(c)-g(0)$ (b) $g(x)+c$ (c) $|g(x)-g(0)|$ (d) $g(c)$ (e) $g''(c)-g''(0)$

43. If $\displaystyle\int_0^k (2kx-x^2)\,dx = 18$, then $k=$

(a) -9 (b) -3 (c) 3 (d) 9 (e) 18

44. Let $F(x)=\displaystyle\int_1^x \sqrt{t^2+2t}\,dt$.

(a) Find $F'(x)$.

(b) Find the domain of F.

(c) Find the length of the curve $y=F(x)$ for $1\le x\le 2$.

45. If f is a continuously differentiable function for all real x, then $\displaystyle\lim_{h\to 0}\frac{1}{h}\int_a^{a+h} f'(x)\,dx$ is

(a) 0 (b) $f(0)$ (c) $f(a)$ (d) $f'(0)$ (e) $f'(a)$

46. Let $a<c<b$ and let g be differentiable on $[a, b]$. Which of the following is NOT necessarily true?

(a) $\displaystyle\int_a^b g(x)\,dx = \int_a^c g(x)\,dx + \int_c^b g(x)\,dx.$

(b) There exists a number d in (a,b) such that $g'(d)=\dfrac{g(b)-g(a)}{b-a}$.

(c) $\displaystyle\int_a^b g(x)\,dx \ge 0.$

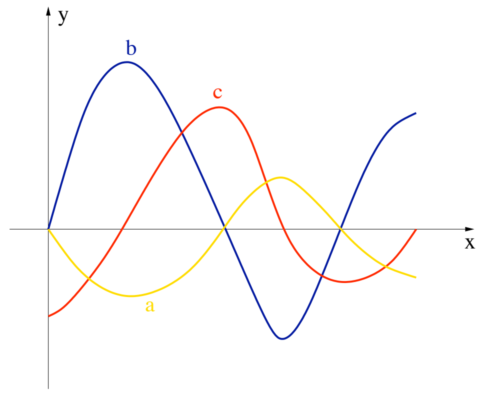
(d) $\displaystyle\lim_{x\to c} g(x)=g(c).$

(e) If k is a real number, then $\displaystyle\int_a^b kg(x)\,dx = k\int_a^b g(x)\,dx$

47. If f is an even and continuous function, i.e., $f(-x)=f(x)$ for all x, then $\displaystyle\int_1^2 f(x)\,dx + \int_{-1}^{-2} f(x)\,dx =$

(a) $2\int_1^2 f(x)\,dx$ (b) $\int_{-1}^1 f(x)\,dx$ (c) 0 (d) $\frac{1}{2}$ (e) None of above

48. The figure shows the graphs of f, f', and $\int_0^x f(t)\,dt$. Identify each graph, and explain your choices.



49. Evaluate the improper integral or show it is divergent.

(a) $\int_0^\infty \frac{1}{(x+2)^4}\,dx$ (b) $\int_1^\infty \frac{\ln x}{x^2}\,dx$ (c) $\int_{-\infty}^0 e^{-2x}\,dx$

(d) $\int_{-1}^1 \frac{1}{2x+1}\,dx$ (e) $\int_1^e \frac{dx}{x\sqrt{\ln x}}$ (f) $\int_2^6 \frac{y}{\sqrt{y-2}}\,dy$

50. A publisher estimates that a book will be sold at the rate of $r(t) = 16,000e^{-0.8t}$ books per year, where t is the number of years from now. Find the total number of books that will ever be sold (up to $t = \infty$).

51. Let R be the shaded region in the first quadrant enclosed by the y axis and the curves of $y = \sin x$ and $y = \cos x$, for $0 \le x \le \pi/4$.

(a) Set up the definite integral for the area of R and evaluate it exactly.

(b) Find the centroid of $(\bar{x}, \bar{y})$ of R.

(c) Set up the integral for the volume of the solid generated when R is revolved about the x axis and evaluate it exactly.

(d) Set up definite integrals to compute the perimeter of R. Do not compute the integrals.

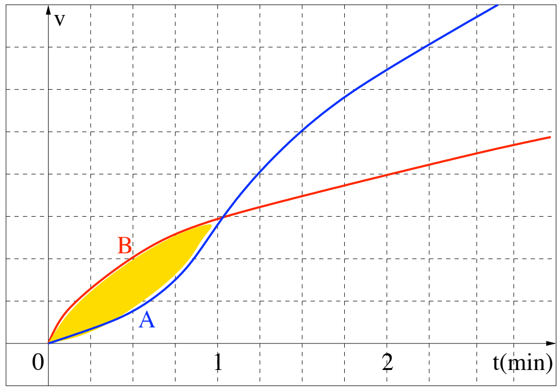
52. Two cars, A and B, start side by side and accelerate from rest. The figure shows the graphs of their velocity functions.

(a) Which car is ahead after one minute? Explain.

(b) What is the meaning of the area of the shaded region?

(c) Which car is ahead after two minutes? Explain.

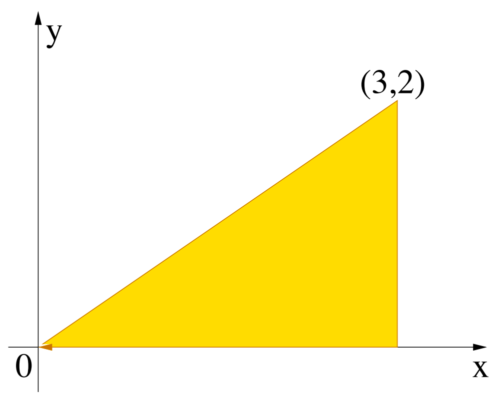
(d) Estimate the time at which the cars are again side by side.



53. Find the area of the region bounded by the given curves.

(a) $y = x^2 - 6x, \quad y = 12x - 2x^2$ (b) $x - 2y + 7 = 0, \quad y^2 - 6y - x = 0$

54. Find the centroid of the region shown.



55. Find the volume of the spherical dome obtained by rotating the region between the graph of $y = \sqrt{R^2 - x^2}$ and the x axis, $R - h \le x \le R$, about the x axis.

56. Let R be the region enclosed by the curves of $y = x$ and $y = \sqrt{x}$. Find the volume of the solid obtained by rotating R about

(a) the x-axis,

(b) the y-axis,

(c) the line $x = -1$,

(d) the line $y = -1$.

57. The amount of pollution in a lake x years ($x \ge 1$) after the closing of a chemical plant is $P(x) = 100/x$ tons. Find the average amount of pollution between 1 and 10 years after the closing.

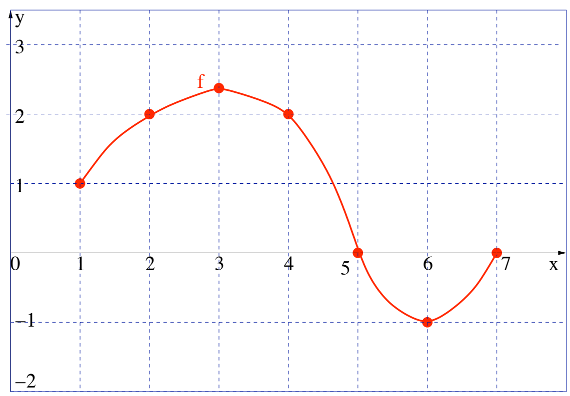
58. Consider the function $f(x) = 1 + x^2$ on the interval [0, 2]. Find a number c in [0, 2] so that the area of the rectangle with base on [0, 2] and height $f(c)$ is equal to the area under the curve of f in the given interval.

59. Compute the length of the curve given by $x = e^t \sin t$ and $y = e^t \cos t$ for $0 \le t \le \pi$.

60. A particle is moved along the x axis by a force that measures $4x^2$ pounds at a point x feet from the origin. Find the work done in moving the particle a distance of 10 feet from the origin.

61. A crane is lifting a 1500 lb transformer from the ground level to the third floor which is 30 feet above the ground level. A 60-foot cable connects the transformer to the top of the crane. The cable weighs 5 lb per foot. How much work is done in lifting the transformer 30 feet above the ground?

62. The graph of a differentiable function f on the closed interval $[1,\,7]$ is shown in the figure. Let $h(x) = \int_1^x f(t)\,dt$ for $1 \le x \le 7$.



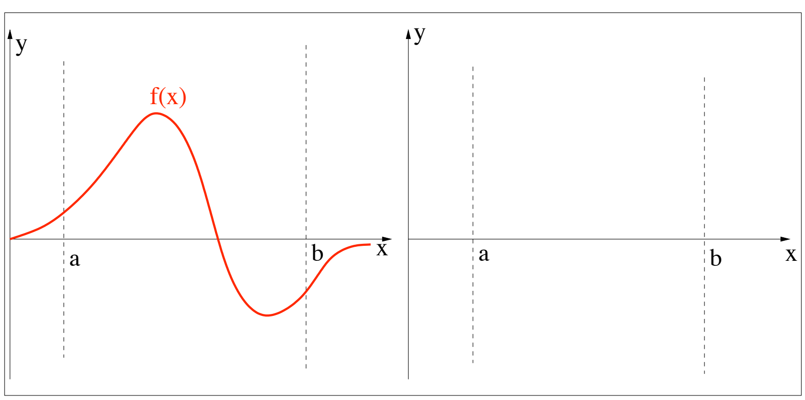
(a) Compute $h(1)$.

(b) Compute $h'(4)$.

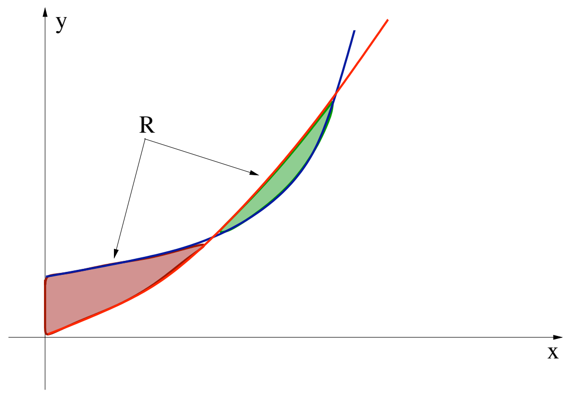
(c) On what interval(s) is the curve of h concave upward? Justify your answers.

(d) Find the value of x at which h has the absolute maximum on the closed interval $[1,\,7]$. Justify your answer.

63. The graph of f is shown below. In the right frame, sketch the graph of $F(x) = \int_a^x f(t)\,dt$ on the interval $[a,\,b]$. Be sure to label the local extrema and inflection points.



64. The figure shows the two shaded regions R enclosed by the curves of $f(x) = x^2$ and $g(x) = 2^x$ in the first quadrant.



(a) Use a calculator to estimate the x coordinates of the two points of intersections of the curves of f and g.

(b) Express the total area of R with definite integrals. (You don't have to evaluate it.)

65. When using the substitution method of integrating, the integral $\int_0^{\pi/2} \sin^3 x \cos x \, dx$ is equal to the integral $\int_0^1 u^3 \, du$ where $u =$

(a) $\cos x$ (b) $-\cos x$ (c) $\sin x$ (d) $-\sin x$ (e) None of these

66. Let $S = \sum_{i=1}^{10} \frac{i^2}{10^3}$ denote a Riemann right-hand sum of $\int_0^1 x^2 \, dx$. Which of the following statements is true?

(a) $S = \int_0^1 x^2 \, dx$ (b) $S < \int_0^1 x^2 \, dx$ (c) $S > \int_0^1 x^2 \, dx$

(d) $S = -\int_0^1 x^2 \, dx$ (e) None of these

67. If $\int x^2(\cos x) \, dx = f(x) - \int (2x)(\sin x) \, dx$, then $f(x) =$

(a) $2\sin x + 2x(\cos x)$ (b) $x^2 \sin x$ (c) $2x(\cos x) - x^2(\sin x)$

(d) $4x \cos x - 2x(\sin x)$ (e) $(2 - x^2)(\cos x) - 4\sin x$.

68. Let $F(x) = \int_0^{x^2} e^{\sin t} \, dt$. Then $F'(\pi) =$

(a) 1 (b) 0 (c) 2π (d) π^2 (e) None of these

69. The average value of $f(x) = \tan^{-1}(x)$ on $[-1, 1]$ is

(a) 0.4388 (b) -0.4388 (c) 0 (d) 0.61562 (e) -0.61562.

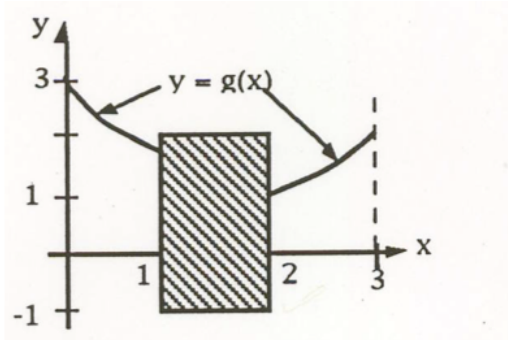
70. $\int_0^1 \tan^{-1} x \, dx =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi - 4\ln 2}{4}$ (c) $\frac{\pi + 4\ln 2}{4}$ (d) $\frac{\pi - 2\ln 2}{4}$ (e) $\frac{\pi + 2\ln 2}{4}$

71. $\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{1 + \sin\theta}} \, d\theta =$

(a) $-2(\sqrt{2} - 1)$ (b) $-2\sqrt{2}$ (c) $2\sqrt{2}$ (d) $2(\sqrt{2} - 1)$ (e) $2(\sqrt{2} + 1)$

72. The function g has a continuous second derivative on the interval $[-1, 4]$. The graph of g is displayed in the figure. The graph of g on the interval $[1, 2]$ is contained in the shaded rectangle. Circle those that are true.



(a) $\int_0^3 g(x)\,dx > 0$ (b) $\int_0^3 g'(x)\,dx > 0$ (c) $\int_0^3 g''(x)\,dx > 0$

73. Consider $f(x) = \dfrac{x^3}{x^2-1}$. Then $f'(x) = \dfrac{x^2(x^2-3)}{(x^2-1)^2}$ and $f''(x) = \dfrac{2x(x^2+3)}{(x^2-1)^3}$.

Find the following with exact answers. You must show all your work to justify your answers.

(a) The intervals where $f(x)$ is increasing and decreasing.

(b) The intervals where $f(x)$ is concave up.

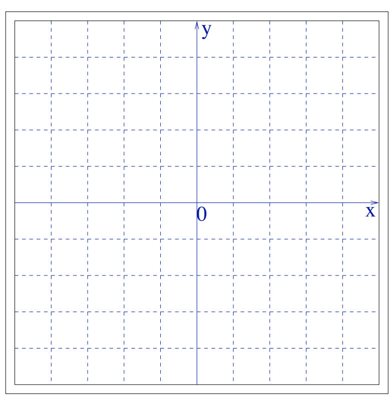
(c) Using the results from (a), find the x coordinates of all relative maximum points.

(d) Using the results from (b) find all inflection points. (Points have two coordinates)

(e) The equations of all horizontal asymptotes. Justify your answer.

(f) The equations of all vertical asymptotes. Justify your answer.

(g) Carefully graph f on the provided coordinate system. Mark all asymptotes with dotted lines, label at least two points.



74. The maximum possible area of a rectangle of perimeter $200m$ is

(a) $2000\,m^2$ (b) $2500\,m^2$ (c) $3500\,m^2$ (d) $2400\,m^2$ (e) $1600\,m^2$

75. Let R be the region in the first quadrant enclosed by the graphs of $y = xe^{-x}$, $x = 0$ and $x = k$ for some $k > 0$. The area of R in terms of k is

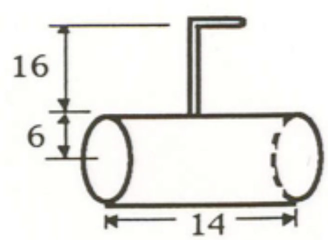
(a) $-ke^{-k} - e^{-k}$ (b) $\frac{1}{2}k^2e^{-k}$ (c) $ke^{-k} - e^{-k} - 1$ (d) $1 - ke^{-k} - e^{-k}$ (e) $ke^{-k} - e^{-k} + 1$

76. A rectangle of perimeter 100 inches is rotated about one of its sides so as to form a cylinder. What are the dimensions of the rectangle which generates a cylinder of maximum volume¿? Justify your answer.

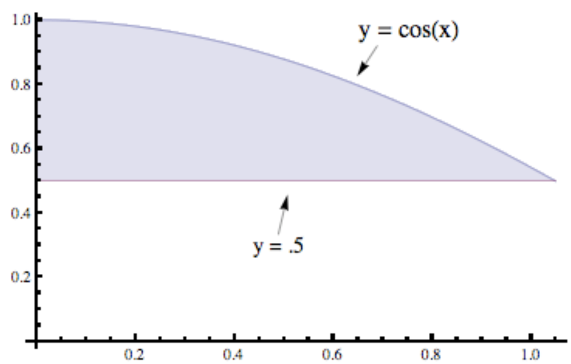
77. A right circular cylinder is to have surface area 150π square inches including both ends and is to enclose the maximum possible volume. What is the height of the cylinder? Justify your answer.

78. A rectangular box with a square base and open top is constructed to have volume of 42,875 cubic inches. The material used to make the bottom costs \$0.06 per square inch and the material used to make the sides costs \$0.03 per square inch. Find the dimensions of the box that minimizes the total costs. Justify your answer.

79. Heating oil for a business complex is stored in a cylindrical tank buried on its side below ground, with the highest part of the tank 16 feet below the surface of the ground. The heating oil, with density ρ in lbs/ft^3 is pumped to an oil furnace located 6 feet above the surface of the ground. The tank is 6 ft in radius and 14 ft long. How much work, in terms of ρ is required in emptying the oil from this tank which is initially full?



80. Let R be the region in the first quadrant enclosed by the y-axis, and the graphs of $\cos x$ and $y = \frac{1}{2}$. Set up the definite integral to compute the volume of the solid generated when R is revolved about the x-axis and use your calculator to approximate it.



81. Find $k > 0$ such that $\int_0^\infty e^{-kt}\, dt = 5$.

82. $\displaystyle\int_2^3 \frac{x}{x^2+1}\, dx =$

(a) $\frac{1}{2}\ln\frac{3}{2}$ (b) $\frac{1}{2}\ln 2$ (c) $2\ln 2$ (d) $\ln 2$ (e) $\frac{1}{2}\ln 5$

83. If $g(x) = \int_0^{x^2} \sqrt{\sin t}\, dt$, then $g'(1) =$

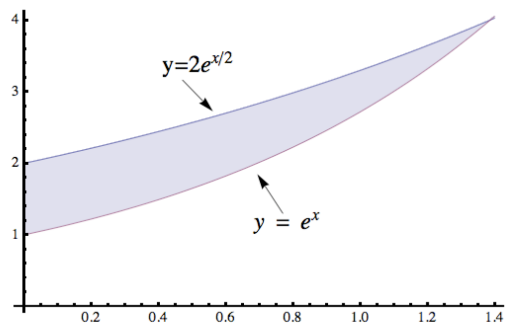
(a) 0.000 (b) 1.132 (c) 1.264 (d) 0.917 (e) 1.835

84. Let h be the function defined by $h(x) = x - \tan^{-1} x$ for all real numbers x. Identify the domain, intercepts, intervals where g is increasing and decreasing, extrema, intervals of concavity, inflection

points and asymptotes. Give all coordinates in exact forms, not decimal approximations. Sketch the graph of $g(x)$ showing all this information and labeling important points.

85. Identify the intervals where $f(x) = e^x + e^{-2x}$ is increasing.

86. Let R be the regions bounded by the y-axis and by the graphs of $y = e^x$ and $y = 2e^{x/2}$.

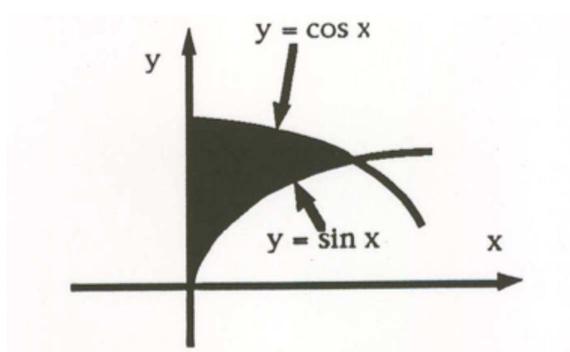


(a) Find the area of R. Set up the integral and evaluate it exactly.

(b) Find the centroid $(\overline{x}, \overline{y})$ of R.

(c) Set up the integral for the volume of the solid generated when R is revolved about the x-axis and evaluate it exactly.

87. Let R be the region in the first quadrant enclosed by the y-axis and the graphs of $y = \sin x$ and $y = \cos x$.



(a) Find the area of R Set up the integral and evaluate it exactly.

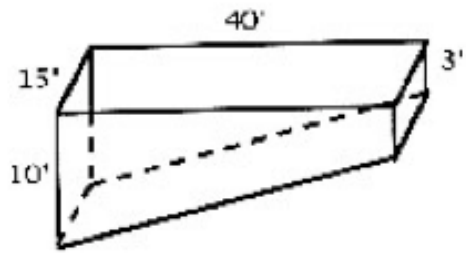
(b) Find the centroid $(\overline{x}, \overline{y})$ of R.

(c) Set up the integral for the volume of the solid generated when R is revolved about the x-axis and evaluate it exactly.

(d) Find the volume of the solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are squares. Set up the integral and evaluate it exactly.

(e) Find the perimeter of R. Set up the definite integrals and use your calculator to approximate them.

88. A swimming pool is 15 feet wide, 40 feet long, 33 feet deep at one end and 10 feet deep at the other end. The pool is filled with water. How much work is required to pump all the water into a drain at the top edge of the pool?



89. The height of an adult man is normally distributed with a mean of 69 inches and a standard deviation of 2.5 inches. How high should a door be so that 90 % of the adult males can pass through without bending?

90. The life of a small motor is normally distributed with an average life of 10 years and a standard deviation of 2 years. The manufacturer replaces, at no charge, all motors that fail while under the warranty. If he is willing to replace at most 3 % of the motors that fail, how long should the warranty be?

91. The length of time for an individual to be served at a cafeteria follows an exponential distribution with a mean of 4 minutes. What is the probability that a person is served in less than 3 minutes?

92. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis is

(a) 2π (b) 4π (c) 6π (d) 9π (e) 12π

93. What is the area of the closed region bounded by the curve $y = e^{2x}$ and the lines $x = 1$ and $y = 1$?

(a) $\dfrac{2-e^2}{2}\sqrt{2}$ (b) $\dfrac{e^2-3}{2}$ (c) $\dfrac{3-e^2}{2}$ (d) $\dfrac{e^2-2}{2}$ (e) $\dfrac{e^2-1}{2}$

94. Find the mean μ for the probability density function $f(x) = \begin{cases} 0 & x < 0 \\ 2x & 0 \le x \le 1 \\ 0 & x > 1 \end{cases}$.

95. The length of the parametric curve given by $x = 2\cos t$, $y = \sin t$ with $0 \le t \le 1$ is

(a) $\int_0^1 (\cos t - 2\sin t)\,dt$ (b) $\int_0^1 \sqrt{3+\cos^2 t}\,dt$ (c) $\int_0^1 \sqrt{3+\sin^2 t}\,dt$
(d) $\int_0^1 \sqrt{1+3\cos^2 t}\,dt$ (e) $\int_0^1 \sqrt{3+\sin^2 t}\,dt$ (f) none of these

96. We are given a function f that is continuous and differentiable on [7, 17]. Suppose we know that $f(7) = 12$ and $f'(x) \le 5$. What is the largest possible value for $f(17)$?

97. For all real numbers a and b, show that $|\cos(2a) - \cos(2b)| \le 2|a - b|$.

98. At 7 p.m., a car is traveling at 50 mph (miles per hour). Ten minutes later, the car has slowed to 30 mph. Show that at some time between 7 and 7:10 the car's acceleration is exactly 120 mph^2.