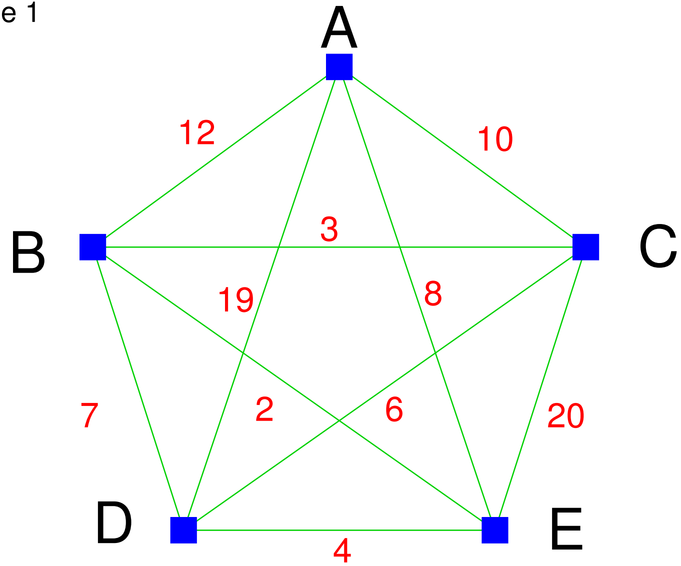
# Examples of Traveling Salesman Problems

- Here are several examples of weighted complete graphs with 5 vertices.
- In each case, we're going to perform the Repetitive Nearest-Neighbor Algorithm and Cheapest-Link Algorithm, then see if the results are optimal.
- Since $N = 5$, $(N-1)! = 24$, so it is feasible to find the optimal Hamiton circuit by brute force (using a computer). But if $N$ were much bigger, then brute force would take too long.
- The point is to see how the RNNA and the CLA compare to brute force.

Example 1



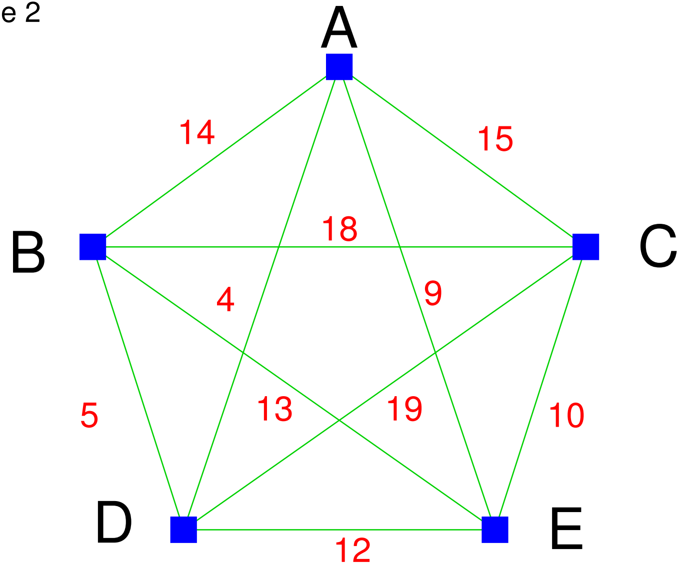
# Results of Example 1

- Output of RNNA: **BEDCAB** (weight **34**)
- Output of CLA: **ACBEDA** (weight **38**)

- In this example, RNNA produces a better result.

- In fact, neither of these Hamilton circuits is optimal – the optimal one is **EACBDE** (weight **32**).

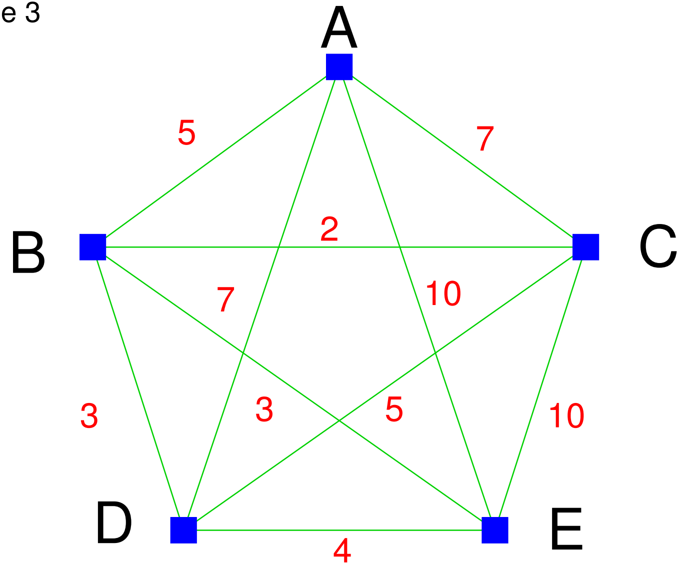
Example 2



# Results of Example 2

- RNNA and CLA both output **DAECBD** (weight **46**)
- This happens to be an optimal Hamilton circuit.

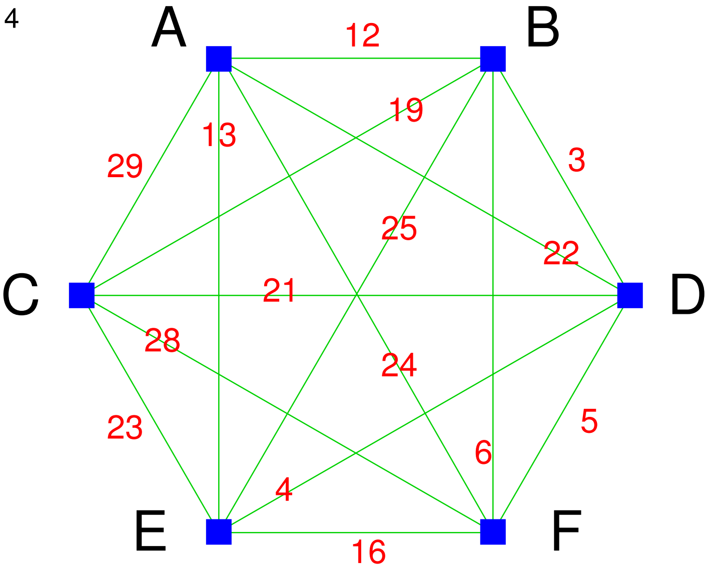
Example 3



# Results of Example 3

- Here, the output of both the CLA and the RNNA may depend on how you break ties. (There's no way to know in advance.)

Example 4



## Distance table for Example 4

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | | 12 | 29 | 22 | 13 | 24 |
| B | 12 | | 19 | 3 | 25 | 6 |
| C | 29 | 19 | | 21 | 23 | 28 |
| D | 22 | 3 | 21 | | 4 | 5 |
| E | 13 | 25 | 23 | 4 | | 16 |
| F | 24 | 6 | 28 | 5 | 16 | |

# Results of Example 4

- Output of RNNA: **FDBAECF** (weight **84**)
- Output of CLA: **ACFBDEA** (weight **83**)

- In this example, CLA produces a better result.

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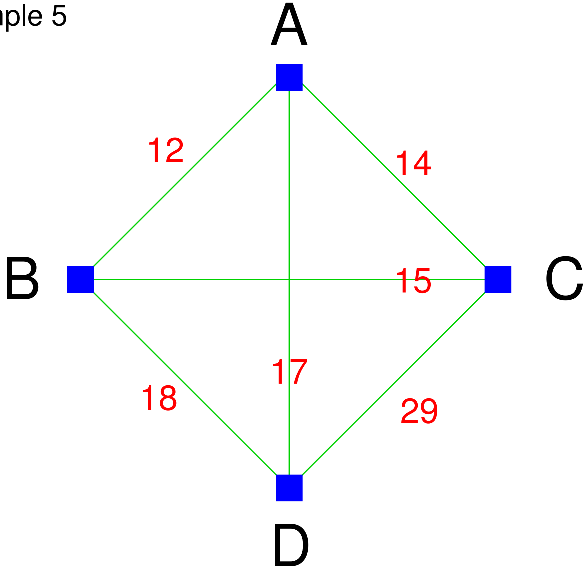
- In this example, CLA produces a better result.

- Neither of these Hamilton circuits is optimal – the optimal one is **FBCAEDF** (weight **76**).

Example 5



# Results of Example 5

| Algorithm | Output | Weight |
|---|---|---|
| NNA (A) | **ABCDA** | $12 + 15 + 29 + 17 =$ **73** |
| NNA (B) | **BACDB** | $12 + 14 + 29 + 18 =$ **73** |
| NNA (C) | CABDC | $= 73$ (same as BACDB) |
| NNA (D) | DABCD | $= 73$ (same as ABCDA) |
| CLA | ABCDA | 73 again |

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- The only other Hamilton circuit in $K_4$ is **ACBDA**, which has weight $14 + 15 + 18 + 17 =$ **64**.

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| CLA | ABCDA | 73 again |

- The only other Hamilton circuit in $K_4$ is **ACBDA**, which has weight $14 + 15 + 18 + 17 =$ **64**.
- So both RNNA and CLA give the **worst possible answer.** (Yuck!)

# The Bad News

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**There is no known algorithm to solve the TSP that is both optimal and efficient.**

- Brute-force is optimal but not efficient.
- NNA, RNNA, and CLA are all efficient but not optimal (and can sometimes produce very bad answers).

- The key word is "**known**." We do not know whether (a) there really is no optimal efficient algorithm, or (b) there really is one and no one has found it yet. Most mathematicians believe (a).