

The Method of Markers: Overview

The **Method of Markers** is a fair-division method for a multiplayer game with discrete goods (e.g., Halloween candy).

In comparison to the Method of Sealed Bids:

- ▶ Everyone gets at least (roughly) a fair share, provided they “bid” honestly.
- ▶ **Disadvantage:** Not suitable if the goods have widely varying values (e.g., an estate)
- ▶ **Advantage:** Doesn't require players to put in cash

The Method of Markers: Example

In Example 3.11 on pp. 98–100 of Tannenbaum, four kids (Alice, Bianca, Carla, Dana) have a pile of Halloween candy that they need to share fairly.

The Method of Markers: Example

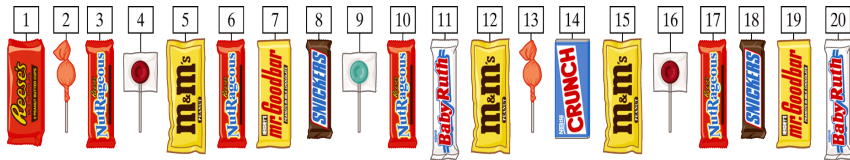
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The Method of Markers: Step 1

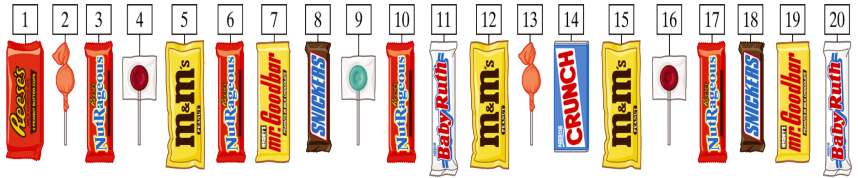
Step 1: Bidding.

The Method of Markers: Step 1

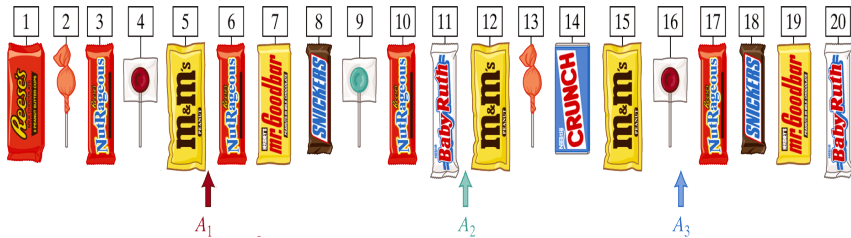
Step 1: Bidding.

Alice places **three markers** A_1 , A_2 , A_3 , dividing the row into **four segments**, each of which she thinks is a fair share.

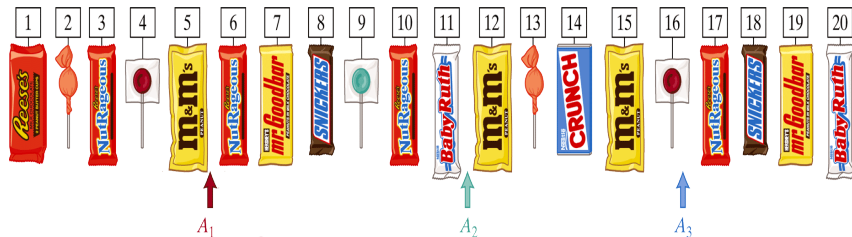
The Method of Markers: Step 1



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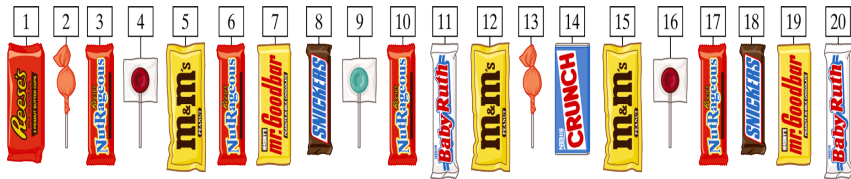


That is, Alice would consider any one of these shares to be fair:

- $\{1, 2, 3, 4, 5\}$ (Alice's 1st segment)
- $\{6, 7, 8, 9, 10, 11\}$ (2nd segment)
- $\{12, 13, 14, 15, 16\}$ (3rd segment)
- $\{17, 18, 19, 20\}$ (4th segment)

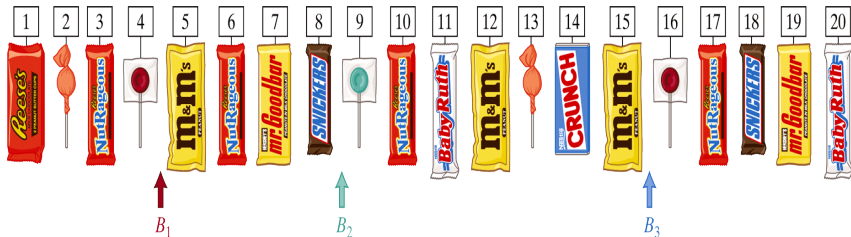
The Method of Markers: Step 1

Similarly, Bianca places markers B_1, B_2, B_3 ; Carla places markers C_1, C_2, C_3 ; Dana places markers D_1, D_2, D_3 .



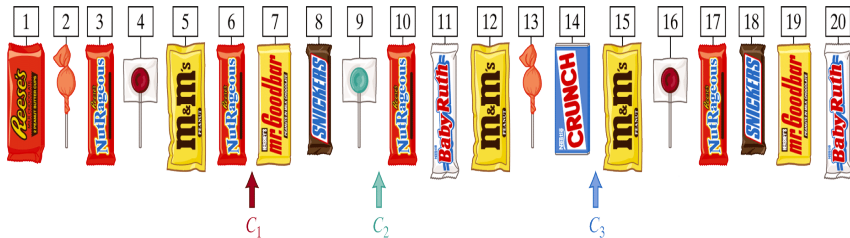
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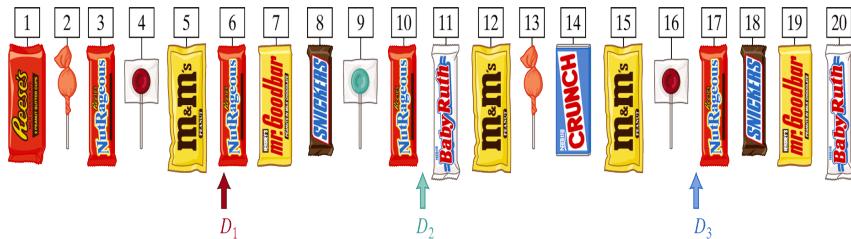
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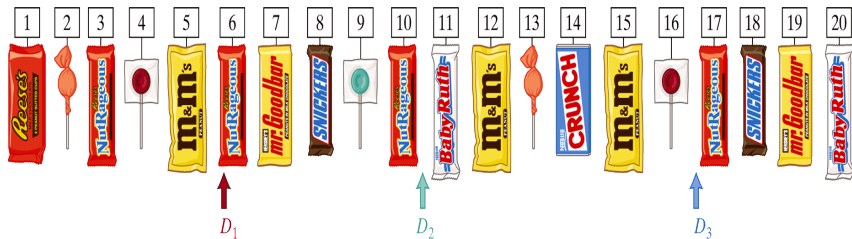
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The Method of Markers: Step 1

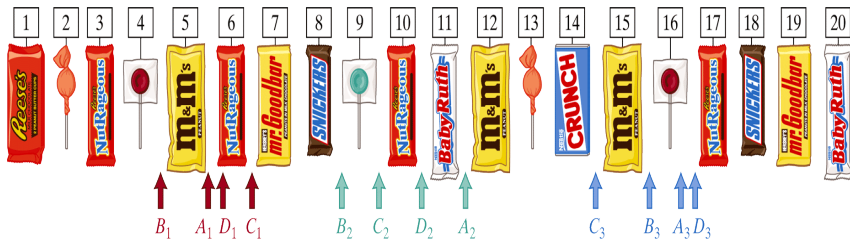
Similarly, Bianca places markers B_1, B_2, B_3 ; Carla places markers C_1, C_2, C_3 ; Dana places markers D_1, D_2, D_3 .



In order to keep the method fair, the players **must all place their markers at the same time**. (For example, they can submit sealed envelopes with the positions of their markers.)

The Method of Markers: Step 1

There are a total of 12 markers (3 for each of 4 players).



Now what?

The Method of Markers: Step 2

Step 2: Allocations.

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Step 2.1: Locate the **leftmost 1st marker** (A_1 , B_1 , C_1 , or D_1).
(If there is a tie, choose one randomly.)

The Method of Markers: Step 2

Step 2: Allocations.

Step 2.1: Locate the **leftmost 1st marker** (A_1 , B_1 , C_1 , or D_1).
(If there is a tie, choose one randomly.)

The corresponding player (here, Bianca) **gets her 1st segment**.

— In this case, Bianca gets

1	2	3	4
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The Method of Markers: Step 2

Step 2: Allocations.

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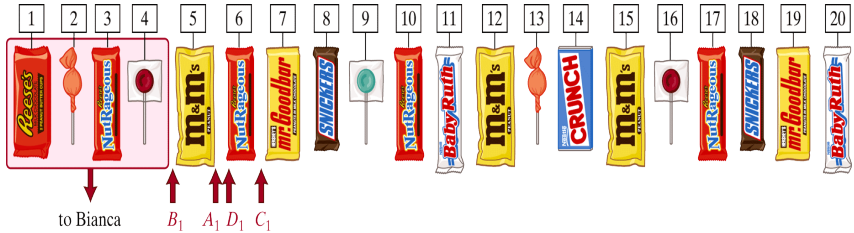
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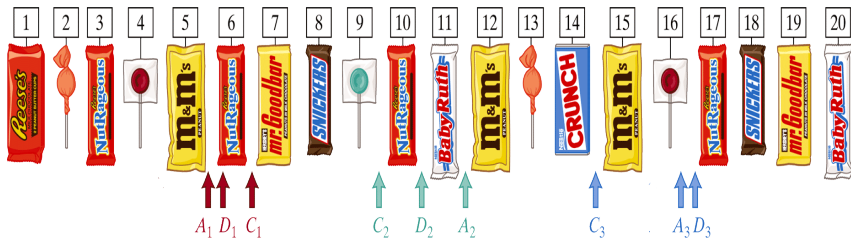
Then, remove all Bianca's markers.

The Method of Markers: Step 2



Bianca's share

The Method of Markers: Step 2



Removing Bianca's markers

The Method of Markers: Step 2

Step 2: Allocations.

Step 2.2: Locate the **leftmost second marker** (A_2 , C_2 , or D_2).

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The corresponding player (here, Carla) **gets her 2nd segment**.

— Carla's share:

7	8	9
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Step 2: Allocations.

Step 2.2: Locate the **leftmost second marker** (A_2 , C_2 , or D_2).

The corresponding player (here, Carla) **gets her 2nd segment**.

— Carla's share:

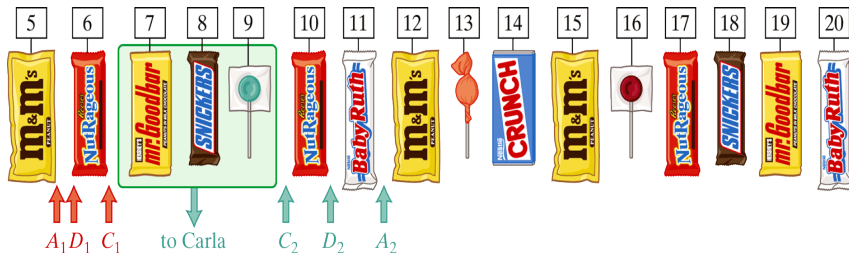
7

8

9

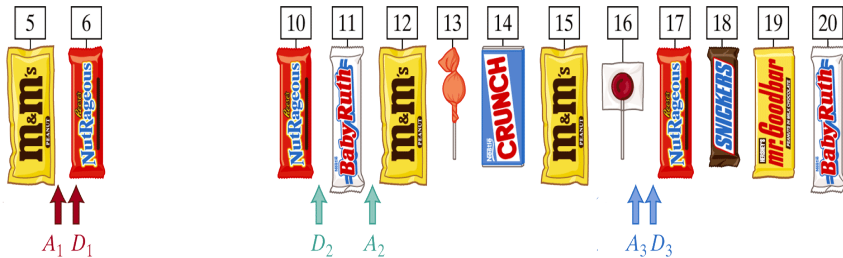
Then, remove all Carla's markers.

The Method of Markers: Step 2



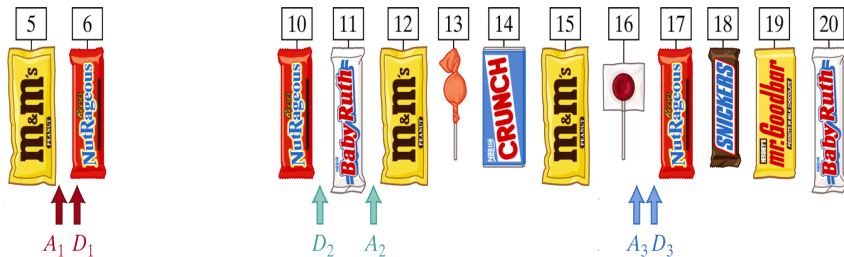
Carla's share (focusing on 1st and 2nd markers)

The Method of Markers: Step 2



Removing Carla's markers

The Method of Markers: Step 2



Note that 5 and 6 have not been allocated to anyone yet.

The Method of Markers: Step 2

Step 2: Allocations.

Step 2.3: Locate the **leftmost third marker** (A_3 or D_3).

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This time, there is a tie; let's say a coin toss chooses Alice rather than Dana.

The Method of Markers: Step 2

Step 2: Allocations.

Step 2.3: Locate the **leftmost third marker** (A_3 or D_3).

This time, there is a tie; let's say a coin toss chooses Alice rather than Dana.

Alice **gets her 3rd segment**.

— Alice's share:

12	13	14	15	16
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The Method of Markers: Step 2

Step 2: Allocations.

Step 2.3: Locate the **leftmost third marker** (A_3 or D_3).

This time, there is a tie; let's say a coin toss chooses Alice rather than Dana.

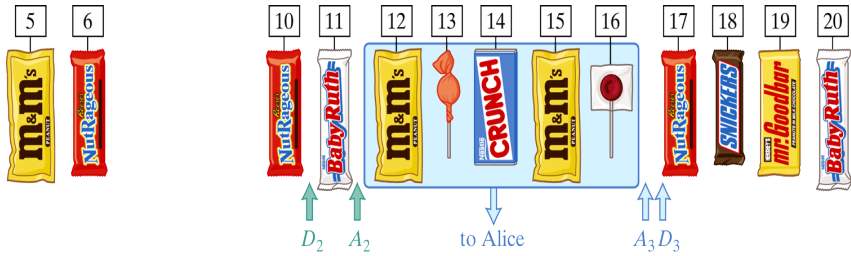
Alice **gets her 3rd segment**.

— Alice's share:

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Then, remove all Alice's markers.

The Method of Markers: Step 2



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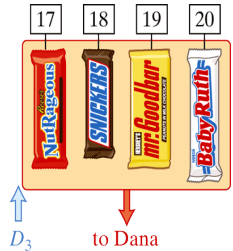
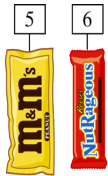
Step 2: Allocations.

Step 2.4: The last player left gets her last segment.

— Dana's share:

17	18	19	20
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The Method of Markers: Step 2



The Method of Markers: Step 3

At this point, everyone has been given a fair share, and usually, there is candy left over!



Step 3: Divide the surplus. (It doesn't matter how — for instance, players get to pick one item at a time in a random order.)

The Method of Markers: Review

Step 0: Arrange the booty to be divided in a row.

Step 1: Bidding. Each of the N players **bids** by placing $N - 1$ markers to separate the booty into N fair shares (in that player's opinion).

(Preserve the Privacy Assumption by having players reveal their bids simultaneously.)

The Method of Markers: Review

Step 2: Allocation.

- ▶ Locate the leftmost **1st marker**.

Give that player his **1st segment** (i.e., from the left end to his 1st marker). Then remove all his markers.

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Give that player his **1st segment** (i.e., from the left end to his 1st marker). Then remove all his markers.

- ▶ Locate the leftmost **2nd marker**.

Give that player her **2nd segment** (i.e., between her 1st and 2nd markers). Then remove all her markers.

The Method of Markers: Review

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Give that player his **1st segment** (i.e., from the left end to his 1st marker). Then remove all his markers.
- ▶ Locate the leftmost **2nd marker**.
Give that player her **2nd segment** (i.e., between her 1st and 2nd markers). Then remove all her markers.
- ▶ Locate the leftmost **3rd marker**.
Give that player her **3rd segment** (i.e., between his 2nd and 3rd markers). Then remove all his markers.
- ...

The Method of Markers: Review

Step 2: Allocation.

- ▶ Eventually, locate the leftmost $(N - 1)^{st}$ marker.
Give that player her $(N - 1)^{st}$ segment
(i.e., between her $(N - 2)^{nd}$ and $(N - 1)^{st}$ markers).
Then remove all her markers.
- ▶ The **last player** gets his **last segment**
(i.e., from his $(N - 1)^{st}$ marker to the end of the row).

The Method of Markers: Review

Step 3: Division of Surplus.

If there are items left over, divide them by, e.g., taking turns choosing one

(or, if there are a **lot** of items, use the Method of Markers all over again!)

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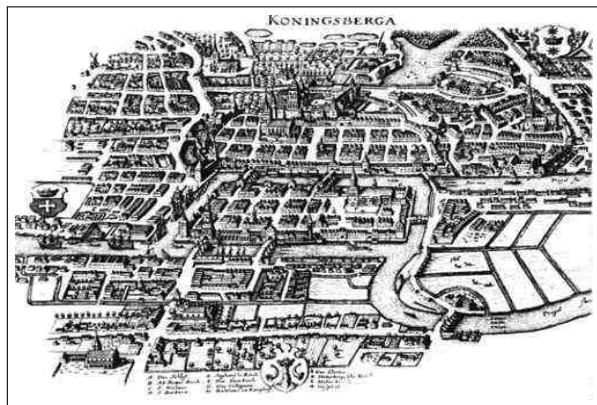
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- ▶ Unlike the Method of Sealed Bids, no cash (or arithmetic!) is required.
- ▶ The players have to be able to divide the booty into roughly equal shares.
- ▶ The method works best if the goods are roughly equivalent in value to each other, and if the players' preferences are fairly close.
(In the example above, what if one of the players is allergic to peanuts?)

And now for something completely different.

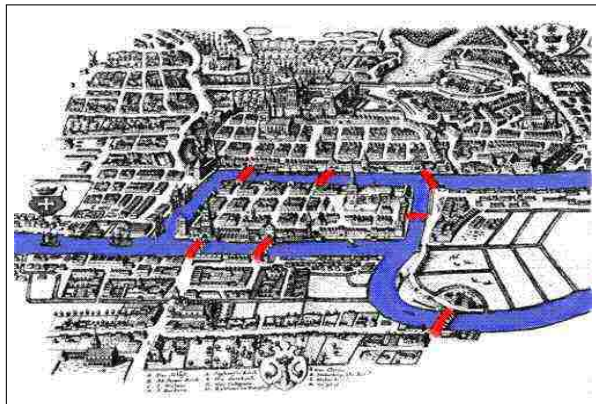
The Seven Bridges of Königsberg

- ▶ In 1735, the city of Königsberg (present-day Kaliningrad) was divided into four districts by the Pregel River.
- ▶ The four districts were connected by seven bridges.

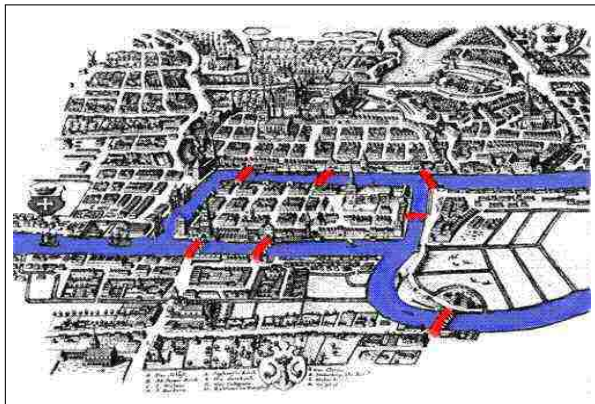
The Seven Bridges of Königsberg



The Seven Bridges of Königsberg



The Seven Bridges of Königsberg



Is it possible to take a walking tour of Königsberg in which you cross each of the seven bridges **exactly once**?