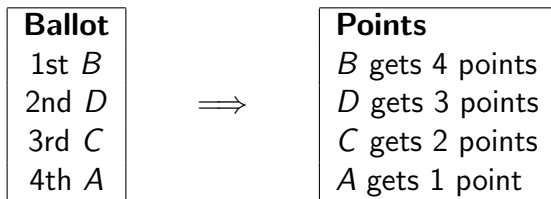


The Borda Count Method (Tannenbaum, §1.3)

The Idea: Award points to candidates based on preference schedule, then declare the winner to be the candidate with the most points.

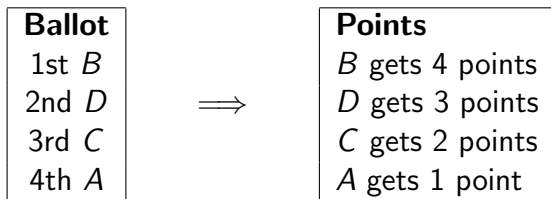
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Ballot 1st <i>B</i> 2nd <i>D</i> 3rd <i>C</i> 4th <i>A</i>	\implies	Points <i>B</i> gets 4 points <i>D</i> gets 3 points <i>C</i> gets 2 points <i>A</i> gets 1 point
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- ▶ Usually attributed to Jean-Charles de Borda (1733–1799), (who was responsible for the metric system!) although first proposed by Ramon Llull (1232–1315).

The Borda Count Method (Tannenbaum, §1.3)

In general, if N is the number of candidates. . .

- ▶ Each first-place vote is worth N points.
- ▶ Each second-place vote is worth $N - 1$ points.
- ▶ Each third-place vote is worth $N - 2$ points.
- ▶ . . .
- ▶ Each N th-place (i.e., last-place) vote is worth 1 point.

Whichever candidate receives the most points wins the election.

A Borda Count Election

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

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For example, Candidate D received

- ▶ 8 first-place votes for $8 \times 4 = 32$ points

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- ▶ 14 fourth-place votes for $14 \times 1 = 14$ points
- ▶ **Total for Candidate D: $32 + 15 + 20 + 14 = 81$ points**

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3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Number of Voters	14	10	8	4	1
1st choice (4 points)	A: 56	C: 40	D: 32	B: 16	C: 4
2nd choice (3 points)	B: 42	B: 30	C: 24	D: 12	D: 3
3rd choice (2 points)	C: 28	D: 20	B: 16	C: 8	B: 2
4th choice (1 point)	D: 14	A: 10	A: 8	A: 4	A: 1

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- ▶ Candidate A: $56+10+8+4+1 = 79$ points
- ▶ Candidate B: $42+30+16+16+2 = 106$ points
- ▶ Candidate C: $28+40+24+8+4 = 104$ points
- ▶ Candidate D: $14+20+32+12+3 = 81$ points

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- ▶ Candidate A: $56+10+8+4+1 = 79$ points
- ▶ Candidate B: $42+30+16+16+2 = 106$ points **Winner!**
- ▶ Candidate C: $28+40+24+8+4 = 104$ points
- ▶ Candidate D: $14+20+32+12+3 = 81$ points

Another Borda Count Election

Example: The Small Seven Athletic Conference would like to invite an eighth member. The possibilities are Alaska (A), Harvard (H), North Dakota (N), and Vermont (V)

All students at SSAC schools are polled. The results:

Percentage of Voters	35%	30%	20%	15%
1st choice	H	V	N	V
2nd choice	N	A	A	N
3rd choice	A	H	H	A
4th choice	V	N	V	H

Who is the winner under the Borda Count?

Facts about the Borda Count

- ▶ Idea behind Borda Count: produce a good compromise candidate (not being hated by voters is just as important as being loved!)

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- ▶ Idea behind Borda Count: produce a good compromise candidate (not being hated by voters is just as important as being loved!)
- ▶ When there are **only two candidates**, the Borda Count **always** produces the same winner as the Plurality Method.
- ▶ Borda Count is susceptible to strategic voting, although in different ways than the Plurality Method.

The Borda Count and Strategic Voting

How might a Borda count election be manipulated?

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Suppose there are two “major-party” candidates (Robbie Republocrat and Debbie Demublican), and one “minor-party candidate” (Izzy Independent).

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- ▶ Izzy’s supporters can now honestly rank Izzy first, rather than having to worry about “wasting their vote”.
- ▶ But Debbie’s supporters may decide, “Robbie is the strongest threat to Debbie, so I should rank Robbie last whether or not I prefer Robbie over Izzy.”

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This is called **“burying”**.

“Burying” (not in Tannenbaum!)

Example: A Borda Count election is held between Robbie, Debbie and Izzy.

Preference Schedule (Version 1)

# Voters	9	7	2	2
1st choice	R	D	I	I
2nd choice	D	R	D	R
3rd choice	I	I	R	D

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# Voters	9	7	2	2
1st choice	R	D	I	I
2nd choice	D	R	D	R
3rd choice	I	I	R	D

- ▶ Debbie's supporters decide to “bury” Robbie by ranking him third instead of second. What happens?

“Burying” (not in Tannenbaum!)

Example: A Borda Count election is held between Robbie, Debbie and Izzy.

Preference Schedule (Version 2)

# Voters	9	7	2	2
1st choice	R	D	I	I
2nd choice	D	I	D	R
3rd choice	I	R	R	D

“Burying” (not in Tannenbaum!)

Example: A Borda Count election is held between Robbie, Debbie and Izzy.

Preference Schedule (Version 2)

# Voters	9	7	2	2
1st choice	R	D	I	I
2nd choice	D	I	D	R
3rd choice	I	R	R	D

- ▶ Robbie's supporters decide to “bury” Debbie by ranking her third. What happens?

“Burying” (not in Tannenbaum!)

Example: A Borda Count election is held between Robbie, Debbie and Izzy.

Preference Schedule (Version 3)

# Voters	9	7	2	2
1st choice	R	D	I	I
2nd choice	I	I	D	R
3rd choice	D	R	R	D

“Burying” (not in Tannenbaum!)

Example: A Borda Count election is held between Robbie, Debbie and Izzy.

Preference Schedule (Version 3)

# Voters	9	7	2	2
1st choice	R	D	I	I
2nd choice	I	I	D	R
3rd choice	D	R	R	D

- ▶ Look who wins now!

The Borda Count and Strategic Voting

“My system is intended only for honest men.”
— *Jean-Charles de Borda*

The Borda Count and Fairness Criteria

Question: Does the Borda Count method satisfy the Majority Criterion?

That is, does receiving a **majority** of the first-place votes guarantee a victory under the Borda Count method? ★

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Answer: **No.**

Let's cook up a preference schedule in which some candidate receives a majority, yet does not win.

Borda Count Fails the Majority Criterion

Number of Voters	60	40
1st choice	A	B
2nd choice	B	D
3rd choice	C	C
4th choice	D	A

Borda Count Fails the Majority Criterion

Number of Voters	60	40			
1st choice	A	B	(4 pts)	A: 240	B: 160
2nd choice	B	D	(3 pts)	B: 180	D: 120
3rd choice	C	C	(2 pts)	C: 120	C: 80
4th choice	D	A	(1 pt)	D: 60	A: 40

- ▶ Candidate A: $240+40 = 280$
- ▶ Candidate B: $180+160 = 340$
- ▶ Candidate C: $120+80 = 200$
- ▶ Candidate D: $60+120 = 180$

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- ▶ Candidate A: $240+40 = 280$ — **Majority candidate**
- ▶ Candidate B: $180+160 = 340$ — **Winner!**
- ▶ Candidate C: $120+80 = 200$
- ▶ Candidate D: $60+120 = 180$

The Borda Count and Fairness Criteria

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Does the Borda Count method satisfy the Condorcet Criterion? That is, if there is a candidate who would beat any other candidate head-to-head, will that candidate always win under the Borda Count method?

Answer: No.

- ▶ Suppose X is a majority candidate.
- ▶ We know that X might not win under Borda Count.
- ▶ But X is definitely a Condorcet candidate!

The Majority Criterion and the Condorcet Criterion

If a voting method does not satisfy the Majority Criterion, then it cannot satisfy the Condorcet Criterion.

- ▶ Reason: If Candidate Q receives a majority of the first-place votes, then Q would beat any other candidate head-to-head, so Q is a Condorcet candidate.

If a voting method satisfies the Condorcet Criterion, then it also satisfies the Majority Criterion.

- ▶ On the other hand, it is possible for a voting method to satisfy the Majority Criterion, but not the Condorcet Criterion.

Plurality vs. Borda Count

	Fairness Criteria		
	Majority	Condorcet	“Public-Enemy”
Plurality	Yes	No	No
Borda Count	No	No	Yes

(“Public-Enemy Criterion”: If a candidate is ranked last by a majority of voters, then s/he *should not* win the election.)