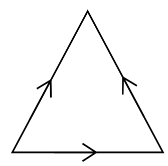
Math 821, Spring 2014
Problem Set #4
Due date: Friday, March 14

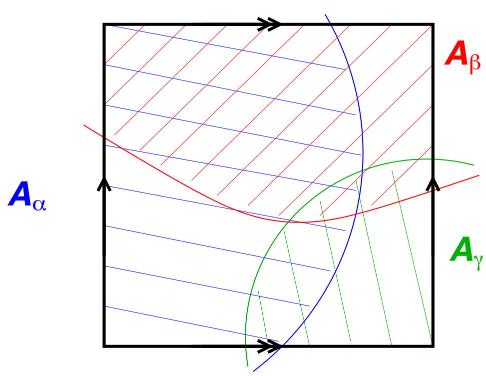
Problem #1 The *dunce hat* is the space D obtained from a triangle by identifying all three edges with each other, with the orientations indicated below.

(#1a) Prove that D is simply-connected using Van Kampen's theorem.
(#1b) Find a different, one-line proof that D is simply-connected.



Problem #2 Consider the standard picture of the torus $T = S^1 \times S^1$ as a quotient space of the square. Explain what is wrong with the following "proof" (whose conclusion is certainly false):

> *Consider the open cover $A_\alpha \cup A_\beta \cup A_\gamma$ shown below. Each open set in the cover is path-connected and simply-connected, and their intersection is path-connected. Therefore, by Van Kampen's theorem, the torus is simply-connected.*



Problem #3 (Hatcher, p.53, #4, modified) Let $n \geq 1$ be an integer, and let $X \subset \mathbb{R}^3$ be the union of n distinct rays emanating from the origin. Compute $\pi_1(\mathbb{R}^3 \setminus X)$.

Problem #4 Let $a_1, \ldots, a_n$ be nonzero integers. Construct a cell complex X from S^1 as follows: For each $j = 1, \ldots, n$, attach a 2-cell to S^1 by wrapping it around the circle a_j times. What is $\pi_1(X)$?

Problem #5 (Hatcher, p.53, #6, modified) Let X be a path-connected cell complex, and let Y be a cell complex obtained from X by attaching an n-cell for some $n \geq 3$. Show that the inclusion $X \hookrightarrow Y$ induces an isomorphism $\pi_1(X) \cong \pi_1(Y)$.

Problem #6 (Hatcher p.79, #2) Show that if $p_1 : \tilde{X}_1 \to X_1$ and $p_2 : \tilde{X}_2 \to X_2$ are covering spaces, then so is their product $p_1 \times p_2 : \tilde{X}_1 \times \tilde{X}_2 \to X_1 \times X_2$.

Problem #7 (Hatcher p.80, #12) Let a and b be the generators of $\pi_1(S^1 \vee S^1, x_0)$ corresponding to the two copies of S^1, with x_0 their common point. Draw a picture of the covering space $\tilde{X}$ of $S^1 \vee S^1$ corresponding to the normal subgroup of $\pi_1(S^1 \vee S^1)$ generated by a^2, b^2, and $(ab)^4$, and prove that this covering space is indeed the correct one. (I.e., this group should be $p_*\pi_1(\tilde{X}, \tilde{x}_0)$.)