Math 725, Spring 2016
Independent Research Project

Project requirements

1. **Read a journal article or book chapter** on a topic in graph theory or something related. Look at the list of topics below, decide which ones seem most appealing to you, and meet with me one-on-one to choose your source. A sample list is provided below; you can choose the article from the list, find an article yourself, or ask me to recommend a source on your favorite topic. (But you must get my approval for the source you have chosen; it shouldn't be too large or too small, and two people cannot do the same project.) Finalize your choice of article as soon as possible, and no later than **Friday, April 8.**

2. **Do something original**, for instance:

- Work out a few examples on your own in order to get the feel for a subject.
- Make a conjecture, and test, prove, and/or disprove it.
- Supply the details of a proof that is sketched or omitted in your primary source.
- If the topic you've chosen involves an algorithm, implement it in the computer language of your choice.

3. **Write a short report** (approximately 3–4 pages, but this may vary) summarizing what you have done. The report is due on **Thursday, April 28.** This is a firm deadline, because the last requirement is...

4. **Read another student's report and give constructive written feedback.** This should take you no more than an hour or two, and the feedback is limited to one page. This report is due on the last day of class: **Thursday, May 6.**

Additional information

Topic Choice: Before you start work, meet with me in person sometime on or before **Friday, April 8** to choose a topic.

Computers: Depending on your topic, you may want to use a computer algebra system. I recommend Sage (www.sagemath.org), which is open-source free software that can be accessed on the Web, or Maple (expensive commercial software, but it is relatively easy to learn and is available on the Math Department computers). If you need a Mathematics Department account, you can obtain one by sending e-mail to help@math.ku.edu stating that you are a student in Math 725, and giving your name and KUID.

Oral Presentations: If you wish to give a short presentation (approximately 20 minutes) in class during the last week of class (Tuesday 5/3 or Thursday 5/5), talk to me. This is not a requirement, as it is not practical to have everyone present, but it is good practice to present, particularly if you are now or plan to be a graduate student in mathematics or a related field.

General instructions on content and style

Your paper should be in narrative style; in particular it should not be just a list of theorems or equations. (It is appropriate, however, to state the major results word-for-word.) It should make sense to a fellow student who has been in the course, but has not read the article. It should give an overview of the article, emphasizing the general ideas and the relationships among the results in the article.

You should include at least one proof. This should not be copied word-for-word from the article. To accomplish this, study the proof until you understand it well. You will probably have to fill in details the author omitted. Then put the article away and write a proof of the result. You should also illustrate special definitions and results with your own examples (different from those in the article).

You should also include a bibliography. This will certainly include the article you are studying, probably a general reference work such as Diestel, and possibly books or articles on related subjects.

Both your report and your evaluation should be composed in LaTeX.

Guidelines on feedback

In your evaluation of your fellow student's project, you should address these questions:

- What did the writer explain as the major themes (e.g., definitions, methods, or theorems) of his or her source?
- What do you now know that you didn't know before?
- What other mathematical ideas came to mind as you read the report?
- What else would you like to know?
- What could the writer have done differently to help you understand the source?

As always when giving constructive criticism to a mathematical colleague, you should be candid, respectful, and specific. The point of the exercise is for both writer and reader to think about what makes good writing. As the critiquer, put yourself in the shoes of the writer reading the comments, and think about what kind of feedback would help you evaluate your own writing and improve it for next time. Here are some examples of comments ranging from helpful to less helpful:

- Helpful: "I would have liked to see more motivation behind the definition of quasiplanar graphs."
- Helpful: "I didn't understand the definition of quasiplanar graphs — what does the term 'pseudo-edge' mean?"
- Only sort of helpful: "I didn't get quasiplanar graphs."
- Not helpful: "Interesting paper, I liked it."

Sample topics (by no means an exhaustive list)

- The Edmonds Blossom Algorithm (finding maximum matchings in not-necessarily-bipartite graphs)
- Applications of Max-Flow/Min-Cut, such as the Gale-Ryser Theorem (characterizing degree sequences of a bipartite graph), and more complex flow problems (multiple sources and sinks, multi-commodity flows, etc.)
- Infinite graphs
- Automorphisms of graphs; isomorphism testing
- Symmetry properties of graphs: regular, strongly regular, vertex-transitive, ...
- Coloring: various chromatic invariants, Vizing's Theorem on edge-colorings, Galvin's Theorem on list-coloring bipartite graphs, ...
- Special classes of graphs: perfect graphs (e.g., Lovász' proof of the Perfect Graph Theorem), threshold graphs (e.g., Merris' theorem on their Laplacian eigenvalues), ...
- Enumeration of spanning trees: alternate proofs and/or generalizations of Cayley's formula, enumeration for special classes of graphs, chip-firing games, ...
- Graphlike structures: matroids, combinatorial geometries, hypergraphs, simplicial complexes, ...
- Random graphs
- Optimization and algorithms (often involving flows, cuts, covers, matchings, and related structures)

- Topological graph theory: genus, crossing number, algorithmic planarity testing, ...
- Algebraic graph theory: eigenvalues...

Some suggested articles

Many of these articles are from the Electronic Journal of Combinatorics (`www.combinatorics.org`), an open-access journal. Some others are available from the preprint server at `www.arXiv.org`. Papers from print journals are mostly available in Anschutz Library, or through interlibrary loan as a last resort.

(1) H.L. Abbott, *Lower bounds for some Ramsey numbers,* Discrete Math. **2** (1972), 289–293.
(2) M.O. Albertson and J.P. Hutchinson, *Graph color extensions: when Hadwiger's Conjecture and embeddings help*, Electronic J. Combin. **9** (2002), no. 1, article #R37.
(3) M. Babenko, A. Gusakov, I. Razenshteyn, *Triangle-Free 2-Matchings Revisited,* arXiv:1003.2697. [A 2-matching is identical to what was called a *bimatching* in problem #5 of HW #4.)
(4) L. W. Beineke, *The decomposition of complete graphs into planar subgraphs*, Graph Theory and Theoretical Physics, Academic Press, New York, 1967, 139–154.
(5) L. W. Beineke and J. W. Moon, *On bipartite tournaments and scores*, The Theory and Applications of Graphs, Wiley, New York, 1981, 55–71.
(6) N.L. Biggs, *Chip-firing and the critical group of a graph*, J. Alg. Combin. **9** (1999), 25–45.
(7) B. Bollobás, L. Pebody, and O. Riordan, *Contraction-deletion invariants for graphs*, J. Combin. Theory Ser. B **80** (2000), no. 2, 320–345.
(8) B. Bollobas and M. Tyomkyn, *Walks and Paths in Trees*, arXiv:1002.2768.
(9) S. A. Burr and P. Erdös, *Generalizations of a Ramsey-theoretic result of Chvátal*, J. Graph Theory **7** (1983), 39–51.
(10) P. Chebotarev and R. Agaev, *Forest matrices around the Laplacian matrix*, Linear Algebra Appl. **356** (2002), 253–274.
(11) P. Cheilaris, B. Keszegh, and D. Palvolgyi, *Unique-maximum and conflict-free colorings for hypergraphs and tree graphs,* arXiv:1002.4210.
(12) J.A. De Loera, C. Hillar, P.N. Malkin, M. Omar, *Recognizing graph-theoretic properties with polynomial ideals*, arXiv:1002.4435. [Requires some knowledge of commutative algebra.]
(13) J. Edmonds, *Minimum partition of a matroid into independent sets*, J. Res. Nat. Bur. Standards **69B** (1965), 67–72.
(14) J. Edmonds and R.M. Karp, *Theoretic improvements in algorithmic efficiency for network flow problems*, J. ACM **19** (1972), 248–264.
(15) R. Ehrenborg and S. van Willigenburg, *Enumerative properties of Ferrers graphs,* Discrete Comput. Geom. **32** (2004), 481–492.
(16) C.D. Godsil, I. Krasikov, and Y. Roditty, *Reconstructing graphs from their k-edge deleted subgraphs*, J. Combin. Theory Ser. B **43** (1987), no. 3, 360–363.
(17) J.L. Gross and S.R. Alpert, *The topological theory of current graphs*, J. Combin. Theory, Series B **17** (1974), 218–233.
(18) R.Häggkvist, R.J. Faudree and R.H. Schelp, *Pancyclic graphs—connected Ramsey number*, Ars Combinatoria **11** (1981), 37–49.
(19) S.L. Hakimi, *On the realizability of a set of integers as degrees of the vertices of a graph*, J. Society for Industrial and Applied Math. **10** (1962), 496–506.
(20) F. Harary and Y. Kodama, *On the genus of an n-connected graph,* Fund. Math. **54** (1964), 7–13.
(21) P. Horák, Q. He, and W.T. Trotter, *Induced matchings in cubic graphs*, *J. Graph Theory* **17** (1993), no. 2, 151–160.
(22) W.-L. Hsu, *Recognizing planar perfect graphs*, J. ACM **34** (1987), 255–288.
(23) A. Idzik, J. Komar and M. Malawski, *Edge-coloured complete graphs: Connectedness of some subgraphs*, Discrete Math. **66** (1987), 119–125.
(24) J.G. Kalbfleisch, *Upper bounds for some Ramsey numbers,* J. Combin. Theory **2** (1967), 35–42.
(25) A.K. Kelmans, *A new planarity condition for 3-connected graphs*, J. Graph Theory **5** (1981), 259–267.

(26) A.K. Kelmans, *Spanning trees of extended graphs*, Combinatorica **12** (1992), 45–51.
(27) J.H. Kim, The Ramsey number $R(3,t)$ has order of magnitude $t^2/\log t$, *Random Structures and Algorithms* **7** (1995), 173–207.
(28) A. King, *Hitting all maximum cliques with a stable set using lopsided independent transversals,* arXiv:0911.1741.
(29) L. Lovász, *Normal hypergraphs and the perfect graph conjecture*, Discrete Math. **2** (1972), 253–267.
(30) J.H. Mason, *On a class of matroids arising from paths in graphs*, Proc. London Math. Soc. (3) **25** (1972), 55–74.
(31) R. Merris, *Degree maximal graphs are Laplacian integral*, Linear Algebra Appl. **199** (1994), 381–389.
(32) C. St. J. A. Nash-Williams, *Hamilton arcs and circuits*, Recent Trends in Graph Theory, Springer-Verlag, 1971, 197–210.
(33) N. Nenov, *Chromatic number of graphs and edge Folkman numbers*, arXiv:1002.4332. [Note: Extremal; bounds involving chromatic number and n(G)]
(34) A. Nilli, *On the second eigenvalue of a graph*, Discrete Math. **91** (1991), 207–210.
(35) E.A. Nordhaus, B.M. Stewart and A.T. White, *On the maximum genus of a graph*, J. Combin. Theory, Series B **11** (1971), 258–267.
(36) K.R. Parthasarathy and G. Ravindra, *The strong perfect graph conjecture is true for $K_{1,3}$-free graphs*, J. Combin. Theory, Series B **22** (1976), 212–223.
(37) P.D. Seymour, *Nowhere-zero 6-flows* J. Combin. Theory Ser. B **30** (1981), 130–135.
(38) P. Seymour and R. Thomas, *An end-faithful spanning tree counterexample,* Discrete Math. **95** (1991), 321-330. [Note: Infinite graphs]
(39) J. Şirán, *End-faithful forests and spanning treees in infinite graphs,* Discrete Math. 95 (1991), 331–340.
(40) R.P. Stanley, *A symmetric function generalization of the chromatic polynomial of a graph*, Adv. Math. **111** (1995), 166–194.
(41) C.M. Terry, L.R. Welch and J.W.T. Youngs, *The genus of $K_n, n = 12(2^m)$*, Bull. Amer. Math. Soc. 71 (1965) 653–656; *The genus of $K_n, n = 12s$*, Bull. Amer. Math. Soc. 71 (1965) 657–660.
(42) C. Thomassen, *Some homeomorphism properties of graphs*, Math. Nachr. **64** (1974), 119–133.
(43) A. Tucker, *The strong perfect graph conjecture for planar graphs*, Canad. J. Math. **25** (1973), 103–114.
(44) J. Tymoczko, *Distinguishing numbers for graphs groups*, Electronic J. Combin. **11** (2004), no. 1, article #R63.
(45) R.M. Wilson, *Decompositions of complete graphs into subgraphs isomorphic to a given graph*, Proceedings of the Fifth British Combinatorial Conference (1975), 647–659.
(46) D.R. Wood and J.A. Telle, *Planar decompositions and the crossing number of graphs with an excluded minor*, arXiv:math.CO/0604467.
(47) D.R. Wood, *A linear upper bound on the rectilinear crossing number*, arXiv:math.CO/0512392.
(48) D.R. Woodall, *Sufficient conditions for circuits in graphs*, Proc. London Math. Soc. **24** (1972), 739–755.
(49) J. Wu and W. Lin, *The strong chromatic index of a class of graphs*, Discrete Math. **308** (2008), no. 24, 6254–6261.
(50) R. Xu and C.-Q. Zhang, *Nowhere-zero 3-flows in squares of graphs*, Electronic J. Combin. **10** (2003), no. 1, article #R5.
(51) F. Zhang and X.-F. Guo, *Hamilton cycles in Euler tour graphs*, J. Combin. Theory, Series B **40** (1986), 1–8.