Math 725, Spring 2010
Independent Research Project

Project requirements

1. Read a journal article or book chapter on a topic in graph theory or something related. You can choose the article from the attached list, find an article yourself, or ask me to recommend an article on your favorite topic. (You must get my approval for the project you have chosen; two people cannot do the same project.)

2. **Do something original**, for instance:

- Work out a few examples on your own in order to get the feel for a subject.
- Make a conjecture, and test, prove, and/or disprove it.
- Supply the details of a proof that is sketched or omitted in your primary source.
- If the topic you've chosen involves an algorithm, implement it in the computer language of your choice.

3. Write a short report (approximately 3–4 pages) summarizing what you've done. The report must be written in good mathematical style (see below) and is due on the last day of class: **Thursday, May 6.**

Additional information

Topic Choice: Before you start work, meet with me in person sometime on or before **Tuesday, April 6** to choose a topic.

Computers: Depending on your topic, you may want to use a computer algebra system; I suggest Sage or Maple. Sage (www.sagemath.org) is open-source free software that can be accessed on the Web. Maple is expensive commercial software, but it is relatively easy to learn and is available on the Math Department computers — if you don't have an account, you can obtain one by sending e-mail to help@math.ku.edu stating that you are a student in Math 725, and giving your name and KUID.

Oral Presentations: If you wish to give a short presentation (approximately 20 minutes) in class during the last week of class (Tuesday 5/4 or Thursday 5/6), talk to me. This is not a requirement, as with 15 students enrolled in the class it is not practical to have everyone present.

General instructions on content and style

Your paper should be in narrative style; in particular it should not be just a list of theorems or equations. (It is appropriate, however, to state the major results word-for-word.) It should make sense to someone who has been in our course, but has not read the article. It should give an overview of the article, emphasizing the general ideas and the relationships among the results in the article.

You should include at least one proof. This should not be copied word-for-word from the article. To accomplish this, study the proof until you understand it well. You will probably have to fill in details the author omitted. Then put the article away and write a proof of the result. You should also illustrate special definitions and results with your own examples (different from those in the article).

You should also include a bibliography. This will certainly include the article you are studying, probably a general reference work such as West's textbook, and possibly books or articles on related subjects.

You should follow the more detailed guidelines on proper mathematical style available from the course website.

You should prepare your report using LaTeX, which is far and away the most widely used system for typesetting mathematical documents—it is easy to use, freely available for all computing platforms, and quite powerful; a working knowledge of LaTeXis almost a must for a professional mathematician. For help getting started, ask a friend (or the instructor), or visit one of the zillion online help sites such as `www.tug.org`.

Sample topics (by no means an exhaustive list)

- Infinite graphs
- Automorphisms of graphs; isomorphism testing
- Symmetry properties of graphs: regular, strongly regular, vertex-transitive, ...
- Coloring: various chromatic invariants, perfect graphs, ...
- Enumeration of spanning trees: alternate proofs and/or generalizations of Cayley's formula, enumeration for special classes of graphs, chip-firing games, ...
- Graphlike structures: matroids, combinatorial geometries, hypergraphs, simplicial complexes, ...
- Random graphs
- Optimization and algorithms (often involving flows, cuts, covers, matchings, and related structures)
- Topological graph theory: genus, crossing number, ...
- Algebraic graph theory: eigenvalues...

Suggested articles

Many of these articles are from the Electronic Journal of Combinatorics (`www.combinatorics.org`), an open-access journal. Some others are available from the preprint server at `www.arXiv.org`. Papers from print journals are mostly available in Anschutz Library, or through interlibrary loan as a last resort.

(1) H.L. Abbott, *Lower bounds for some Ramsey numbers,* Discrete Math. **2** (1972), 289–293.
(2) M.O. Albertson and J.P. Hutchinson, *Graph color extensions: when Hadwiger's Conjecture and embeddings help*, Electronic J. Combin. **9** (2002), no. 1, article #R37.
(3) M. Babenko, A. Gusakov, I. Razenshteyn, *Triangle-Free 2-Matchings Revisited,* arXiv:1003.2697. [A 2-matching is identical to what was called a *bimatching* in problem #5 of HW #4.)
(4) L. W. Beineke, *The decomposition of complete graphs into planar subgraphs*, Graph Theory and Theoretical Physics, Academic Press, New York, 1967, 139–154.
(5) L. W. Beineke and J. W. Moon, *On bipartite tournaments and scores*, The Theory and Applications of Graphs, Wiley, New York, 1981, 55–71.
(6) F. Bernhart, *A digest of the four color theorem*, J. Graph Theory **1** (1977), 207–225.
(7) N.L. Biggs, *Chip-firing and the critical group of a graph*, J. Alg. Combin. **9** (1999), 25–45. **Claimed by Derek**
(8) B. Bollobás, L. Pebody, and O. Riordan, *Contraction-deletion invariants for graphs*, J. Combin. Theory Ser. B **80** (2000), no. 2, 320–345.
(9) Louis J. Billera, Ning Jia, and Victor Reiner, *A quasisymmetric function for matroids*, European J. Combin. 30 (2009), 1727–1757. **Claimed by Chris**
(10) Bela Bollobas and Mykhaylo Tyomkyn, *Walks and Paths in Trees*, arXiv:1002.2768.

(11) S. A. Burr and P. Erdös, *Generalizations of a Ramsey-theoretic result of Chvátal*, J. Graph Theory **7** (1983), 39–51.
(12) P. Chebotarev and R. Agaev, *Forest matrices around the Laplacian matrix*, Linear Algebra Appl. **356** (2002), 253–274.
(13) Panagiotis Cheilaris, Balazs Keszegh, and Domotor Palvolgyi, *Unique-maximum and conflict-free colorings for hypergraphs and tree graphs,* arXiv:1002.4210.
(14) J.A. De Loera, C. Hillar, P.N. Malkin, M. Omar, *Recognizing graph-theoretic properties with polynomial ideals*, arXiv:1002.4435. [Requires some knowledge of commutative algebra.]
(15) J. Edmonds, *Minimum partition of a matroid into independent sets*, J. Res. Nat. Bur. Standards **69B** (1965), 67–72. **Claimed by Sarvesh**
(16) J. Edmonds and R.M. Karp, *Theoretic improvements in algorithmic efficiency for network flow problems*, J. ACM **19** (1972), 248–264.
(17) Richard Ehrenborg and Stephanie van Willigenburg, *Enumerative properties of Ferrers graphs,* Discrete Comput. Geom. **32** (2004), 481–492.
(18) C.D. Godsil, I. Krasikov, and Y. Roditty, *Reconstructing graphs from their k-edge deleted subgraphs*, J. Combin. Theory Ser. B **43** (1987), no. 3, 360–363.
(19) J.L. Gross and S.R. Alpert, *The topological theory of current graphs*, J. Combin. Theory, Series B **17** (1974), 218–233.
(20) R.Häggkvist, R.J. Faudree and R.H. Schelp, *Pancyclic graphs—connected Ramsey number*, Ars Combinatoria **11** (1981), 37–49.
(21) S.L. Hakimi, *On the realizability of a set of integers as degrees of the vertices of a graph*, J. Society for Industrial and Applied Math. **10** (1962), 496–506.
(22) F. Harary and Y. Kodama, *On the genus of an n-connected graph,* Fund. Math. **54** (1964), 7–13.
(23) P. Horák, Q. He, and W.T. Trotter, *Induced matchings in cubic graphs*, *J. Graph Theory* **17** (1993), no. 2, 151–160.
(24) Wen-Lian Hsu, *Recognizing planar perfect graphs*, J. ACM **34** (1987), 255–288.
(25) Adam Idzik, Jan Komar and Marcin Malawski, *Edge-coloured complete graphs: Connectedness of some subgraphs*, Discrete Math. **66** (1987), 119–125.
(26) J.G. Kalbfleisch, *Upper bounds for some Ramsey numbers,* J. Combin. Theory **2** (1967), 35–42.
(27) A.K. Kelmans, *A new planarity condition for 3-connected graphs*, J. Graph Theory **5** (1981), 259–267.
(28) A.K. Kelmans, *Spanning trees of extended graphs*, Combinatorica **12** (1992), 45–51. **Claimed by Jon**
(29) J.H. Kim, The Ramsey number $R(3,t)$ has order of magnitude $t^2/\log t$, *Random Structures and Algorithms* **7** (1995), 173–207.
(30) Andrew King, *Hitting all maximum cliques with a stable set using lopsided independent transversals,* arXiv:0911.1741.
(31) L. Lovász, *Normal hypergraphs and the perfect graph conjecture*, Discrete Math. **2** (1972), 253–267.
(32) J.H. Mason, *On a class of matroids arising from paths in graphs*, Proc. London Math. Soc. (3) **25** (1972), 55–74.
(33) R. Merris, *Degree maximal graphs are Laplacian integral*, Linear Algebra Appl. **199** (1994), 381–389.
(34) C. St. J. A. Nash-Williams, *Hamilton arcs and circuits*, Recent Trends in Graph Theory, Springer-Verlag, 1971, 197–210.
(35) N. Nenov, *Chromatic number of graphs and edge Folkman numbers*, arXiv:1002.4332. [Note: Extremal; bounds involving chromatic number and n(G)]
(36) A. Nilli, *On the second eigenvalue of a graph*, Discrete Math. **91** (1991), 207–210.
(37) E.A. Nordhaus, B.M. Stewart and A.T. White, *On the maximum genus of a graph*, J. Combin. Theory, Series B **11** (1971), 258–267.
(38) K.R. Parthasarathy and G. Ravindra, *The strong perfect graph conjecture is true for $K_{1,3}$-free graphs*, J. Combin. Theory, Series B **22** (1976), 212–223.
(39) P.D. Seymour, *Nowhere-zero 6-flows* J. Combin. Theory Ser. B **30** (1981), 130–135.
(40) P. Seymour and R. Thomas, *An end-faithful spanning tree counterexample,* Discrete Math. **95** (1991), 321-330. [Note: Infinite graphs]
(41) Jozef Şirán, *End-faithful forests and spanning treees in infinite graphs,* Discrete Math. 95 (1991), 331–340.

(42) R.P. Stanley, *Acyclic orientations of graphs*, Discrete Math. **5** (1973), 171–178. **Claimed by Jana**
(43) R.P. Stanley, *A symmetric function generalization of the chromatic polynomial of a graph*, Adv. Math. **111** (1995), 166–194. **Claimed by Alex**
(44) C.M. Terry, L.R. Welch and J.W.T. Youngs, *The genus of* $K_n, n = 12(2^m)$, Bull. Amer. Math. Soc. 71 (1965) 653–656; *The genus of* $K_n, n = 12s$, Bull. Amer. Math. Soc. 71 (1965) 657–660. **Claimed by C.J.**
(45) C. Thomassen, *Some homeomorphism properties of graphs*, Math. Nachr. **64** (1974), 119–133.
(46) A. Tucker, *The strong perfect graph conjecture for planar graphs*, Canad. J. Math. **25** (1973), 103–114.
(47) J. Tymoczko, *Distinguishing numbers for graphs groups*, Electronic J. Combin. **11** (2004), no. 1, article #R63.
(48) R.M. Wilson, *Decompositions of complete graphs into subgraphs isomorphic to a given graph*, Proceedings of the Fifth British Combinatorial Conference (1975), 647–659.
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(50) David R. Wood, *A linear upper bound on the rectilinear crossing number*, arXiv:math.CO/0512392.
(51) D.R. Woodall, *Sufficient conditions for circuits in graphs*, Proc. London Math. Soc. **24** (1972), 739–755.
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