Math 724, Fall 2013
Homework #6

Instructions: Write up your solutions in LaTeX and hand in a hard copy in class on **Friday, November 22.** Collaboration is allowed (and in fact encouraged), but each student must write up his or her solutions independently and acknowledge all collaborators.

Problem #1 Consider the sequence $T_0, T_1, T_2, T_3, \ldots$ defined recursively by

$$T_n = T_{n-1} + T_{n-2} + T_{n-3} \qquad \text{for } n \geq 3.$$

Find a closed-form expression (in terms of x, T_0, T_1, T_2) for the generating function

$$\Omega = \sum_{n \geq 0} T_n x^n.$$

Problem #2 Give a combinatorial interpretation for the coefficient of $q^k x^\ell$ in the power series

$$\prod_{n=1}^{\infty} (1 + qx^n + qx^{2n} + qx^{3n} + qx^{4n} + \cdots).$$

Problem #3 Bogart #224.

Problem #4 Bogart, Chapter 4, Supplementary Problem #4.

Problem #5 Bogart, Chapter 4, Supplementary Problem #9.

Problem #6 Bogart #234.

Problem #7 Bogart #238 and #239. (Once you do #238, problem #239 should be easy.)

Problem #8 The game of *egdirb* uses a deck of 30 cards. There are three suits: artichokes, ferrets, and pumpkins. Each suit contains ten cards. In one deal of egdirb, each of three players (Larry, Curly and Moe) receives a hand of 10 cards. Use inclusion/exclusion to determine the probability that at least one player is dealt a void (i.e., zero cards) in at least one suit.

Extra credit: Redo problem #7 for the game of bridge (with a standard four-suited deck and four players, each of whom receives 13 cards). It is OK to write a computer program (preferably in Sage) to compute the answer; if you do so, include the source code in your written solutions.