Math 243, Fall 2015
Final Exam Review Information

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1. Logistics. The exam will be on **Tuesday, December 15, 10:30am-1:00pm, in Snow 564** (the regular classroom). Bring a supply of loose-leaf paper to the test. You may bring a calculator, but you will not need one.

I will hold office hours (Snow 618) on Friday, December 11 (Stop Day), 9am–12pm, and Monday, December 14, 10am–12pm and 2pm–4pm. As always, I am happy to answer questions by e-mail.

The test will be approximately one and a half times as long as the midterm tests. On the other hand, you will have three times as much time to do it. About 80% of the points on the test will come from problems similar to the regular homework problems. The remaining 20% will be harder, more similar to the honors problems.

Please check the KU website for updates in case of inclement weather. The Office of the Provost asks instructors to inform students that, per University policy, no final examination will be canceled or postponed to a later date because of a building evacuation.

2. Topics. The exam will cover the entire semester, with a strong focus on material from chapters 6 and 7. For earlier material, please refer to the review handouts for the two midterm tests. Here are specific topics and suggested review problems for the last two chapters.

- Scalar and vector line integrals: definition, different notations, how to evaluate them, and reparametrizations (§6.1: #1–40)
- Basic application of Green's Theorem: usage and application to finding areas (§6.2: #1–18)
- Conservative vector fields; finding scalar potentials (§6.3: #1–18, 26–28)
- Topological aspects of Green's Theorem, including simply-connected regions and winding numbers (see my notes in the "More Resources" section of Blackboard)

- Surfaces to know how to parametrize: plane, disk, parallelogram, sphere, cylinder, cone (throughout chapter 7)
- Given a parametrized surface $\mathbf{X} : D \subset \mathbb{R}^2 \to \mathbb{R}^3$, find coordinate curves, tangent vectors, normal vectors and tangent planes, and express the surface as the graph of a function $f : \mathbb{R}^2 \to \mathbb{R}$ or as a level surface of a function $f : \mathbb{R}^3 \to \mathbb{R}$ (§7.1: #1–21; §7.Misc: #1–8)
- Find the surface area of a parametrized surface or of the graph of a function $f : \mathbb{R}^2 \to \mathbb{R}$ (§7.1: #22–29)
- Set up and evaluate a scalar or vector surface integral over a parametrized surface in $\mathbb{R}^3$, and understand the interpretation of a vector surface integral in terms of flux (§7.2: #1–28)
- Know what happens to a surface integral when the surface is reparametrized (§7.2: #4)
- Verify Stokes' Theorem for a given surface and vector field (§7.3: #1–5)
- Verify Gauss's Theorem for a given closed surface and vector field (§7.3: #6–9)
- Applications of Stokes' and Gauss' Theorems, including replacing a difficult integral with easier ones (§7.3: #11–18, 20; §7.Misc: 21–26, 28)

3. Formulas. The following formulas will be provided to you on the test. You don't have to memorize them, but you do need to know how and when to use them and what the notation means. (If you are working on a review problem and you need a formula that is not on the list below, then that means that you need to know it.)

Rectangular and spherical coordinates in $\mathbb{R}^3$:

$$\begin{array}{lll} x = \rho \sin\varphi \cos\theta & \rho^2 = x^2 + y^2 + z^2 & 0 \leqslant \rho \\ y = \rho \sin\varphi \sin\theta & \tan\varphi = \sqrt{x^2+y^2}/z & 0 \leqslant \varphi \leqslant \pi \\ z = \rho \cos\varphi & \tan\theta = y/x & 0 \leqslant \theta \leqslant 2\pi \end{array}$$

Rectangular and hyperspherical coordinates in $\mathbb{R}^4$:

$$\begin{array}{lll} w = \rho \sin\varphi_1 \sin\varphi_2 \cos\varphi_3 & \rho = \sqrt{w^2+x^2+y^2+z^2} & 0 \leqslant \rho \\ x = \rho \sin\varphi_1 \sin\varphi_2 \sin\varphi_3 & \tan\varphi_1 = \sqrt{w^2+x^2+y^2}/z & 0 \leqslant \varphi_1 \leqslant \pi \\ y = \rho \sin\varphi_1 \cos\varphi_2 & \tan\varphi_2 = \sqrt{w^2+x^2}/y & 0 \leqslant \varphi_2 \leqslant \pi \\ z = \rho \cos\varphi_1 & \tan\varphi_3 = w/x & 0 \leqslant \varphi_3 \leqslant 2\pi \end{array}$$

Curvature and torsion:

$$\kappa = \left\| \frac{d\mathbf{T}}{ds} \right\| = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|^3} \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}; \qquad \tau = \frac{(\mathbf{v} \times \mathbf{a}) \cdot \mathbf{a}'}{\|\mathbf{v} \times \mathbf{a}\|^2}$$

Line integrals:

$$\int_{\mathbf{x}} f\, ds = \int_a^b f(\mathbf{x}(t))\, \|\mathbf{x}'(t)\|\, dt \qquad \int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{x}(t)) \cdot \mathbf{x}'(t)\, dt$$

Green's Theorem:

$$\oint_{\partial D} \mathbf{F} \cdot d\mathbf{s} = \iint_D (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dA \qquad \text{or} \qquad \oint_{\partial D} M\, dx + N\, dy = \iint_D (N_x - M_y)\, dx\, dy$$

Winding numbers: For a oriented closed curve $\mathbf{x}$ that does not pass through the origin,

$$w(C, \mathbf{0}) \;=\; \frac{1}{2\pi} \oint_C \frac{-y\mathbf{i} + x\mathbf{j}}{x^2+y^2} \cdot d\mathbf{s} \;=\; \frac{1}{2\pi} \oint_C \frac{-y\, dx + x\, dy}{x^2+y^2}.$$

Surface integrals:

$$\iint_{\mathbf{X}} f\, dS = \iint_D f(\mathbf{X}(s,t))\, \|\mathbf{N}(s,t)\|\, ds\, dt \qquad \iint_{\mathbf{X}} \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F}(\mathbf{X}(s,t)) \cdot \mathbf{N}(s,t)\, ds\, dt$$

Stokes' Theorem:

$$\iint\limits_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \oint\limits_{\partial S} \mathbf{F} \cdot d\mathbf{s}$$

Gauss's Theorem (a.k.a. the Divergence Theorem):

$$\iiint\limits_B (\nabla \cdot \mathbf{F})\, dV = \oiint\limits_{\partial B} \mathbf{F} \cdot d\mathbf{S}$$

Volume and hypervolume elements:

$$\begin{aligned}
(\mathbb{R}^3) \qquad dV &= \rho^2 \sin\varphi \; d\rho \; d\varphi \; d\theta \\
&= \rho \; d\rho \; d\theta \; dz \\
(\mathbb{R}^4) \qquad dV &= \rho^3 \sin^2\!\varphi_1 \sin\varphi_2 \; d\rho \; d\varphi_1 \; d\varphi_2 \; d\varphi_3
\end{aligned}$$

Some useful integrals (others may be supplied if necessary):

$$\int \sin^2 u \; du = \frac{u - \sin u \cos u}{2} + C \qquad\qquad \int_0^{2\pi} \sin^2 u \; du = \pi$$

$$\int \cos^2 u \; du = \frac{u + \sin u \cos u}{2} + C \qquad\qquad \int_0^{2\pi} \cos^2 u \; du = \pi$$