

Math 243, Fall 2015
Extra Honors Problem for HW #8

Can you come up with a C^∞ vector field $\mathbf{F} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ with the property that $\operatorname{div} \mathbf{F}(x, y) \geq 0$ if $x \geq 0$ or $y \geq 0$, but $\operatorname{div} \mathbf{F}(x, y) < 0$ if $x < 0$ and $y < 0$? (I.e., the divergence is zero on the x - and y -axes, positive in the open first, second and fourth quadrants, and negative in the open third quadrant.) Or can you prove that no such vector field exists?

(This question was asked by a student in Math 223 in Fall 2010, sparking a lively debate in class.)