Math 223, Fall 2012
Review Information for Test #2

Logistics. The test will be in class on **Friday 11/9/12**. Wednesday's class (11/7/12) will be a Q&A session. Bring a supply of loose-leaf paper to the test. You may bring a calculator.

Topics. The focus of the test will be material from lectures and homeworks through Monday, November 8 (essentially chapters 3 and 6 of Colley). Specific topics to know and sample problems:

- Parametrized curves, velocity, acceleration and speed (§3.1, problems 1–10, 15–32)
- Arclength and the arclength parameter (§3.2, problems 1–16)
- The unit tangent, normal and binormal vectors; curvature and torsion (§3.2, problems 17–26, 35–42)
- Vector fields, gradient fields, potentials, flow lines (§3.3, problems 1–24)
- Gradient, divergence, curl, what they mean geometrically, and the ∇ operator (§3.4, problems 1–26)
- Additional problems involving topics of chapter 3. (True/False questions on p.237 are good review. Also, many of the problems on pp.237–243.)
- Scalar and vector line integrals: definition, different notations, how to evaluate them, and reparametrizations (§6.1, problems 1–40)
- Basic application of Green's Theorem: usage and application to finding areas (§6.2, problems 1–18)
- Conservative vector fields; finding scalar potentials (§6.3, problems 1–18, 26–28)

Obviously, doing all the problems listed above is impractical. In order to study for the test, do a few problems on each topic. If it seems easy, go on to the next topic. If it seems hard, then do more problems on it!

You are also responsible for all material covered on the first midterm test (chapters 1–2 of Colley) and in Math 122 (e.g., double integrals) that is necessary for the topics above.

Topics in chapters 3 and 6 of Colley that you are *not* responsible for:

- Kepler's laws
- Tangential/normal components of acceleration
- Frenet-Serret formulas
- Other coordinate formulation of the *nabla* operator
- Numerical evaluation of line integrals
- Divergence theorem in the plane

Formulas. The following formulas will be provided to you on the test. You don't have to memorize them, but you do need to know how and when to use them and what the notation means. (If you are working on a review problems and you need a formula that is not on the list below, then that means that you need to know that formula!)

Conversion between rectangular and spherical coordinates:

$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases} \qquad \begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \tan\phi = \sqrt{x^2 + y^2}/z \\ \tan\theta = y/x \end{cases}$$

Product and Quotient Rules:

If $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}$ are differentiable at $\mathbf{a} \in \mathbb{R}^n$, then

$$D(fg)(\mathbf{a}) = g(\mathbf{a})Df(\mathbf{a}) + f(\mathbf{a})Dg(\mathbf{a}),$$

$$D(f/g)(\mathbf{a}) = \frac{g(\mathbf{a})Df(\mathbf{a}) - f(\mathbf{a})Dg(\mathbf{a})}{g(\mathbf{a})^2} \qquad \text{(provided that } g(\mathbf{a}) \neq 0)$$

Chain Rule: If $g : \mathbb{R}^n \to \mathbb{R}^m$ and $f : \mathbb{R}^m \to \mathbb{R}^k$ are functions such that g is differentiable at $\mathbf{a} \in \mathbb{R}^n$, and f is differentiable at $g(\mathbf{a}) \in \mathbb{R}^m$, then

$$D(f \circ g)(\mathbf{a}) = \big[Df(g(\mathbf{a}))\big]\,\big[Dg(\mathbf{a})\big]$$

Curvature:

$$\kappa = \left\| \frac{d\mathbf{T}}{ds} \right\| = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|^3}$$

Torsion:

$$\frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}; \qquad \tau = \frac{(\mathbf{v} \times \mathbf{a}) \cdot \mathbf{a}'}{\|\mathbf{v} \times \mathbf{a}\|^2}$$

Line integrals:

$$\int_{\mathbf{x}} f\, ds = \int_a^b f(\mathbf{x}(t))\, \|\mathbf{x}'(t)\|\, dt \qquad \int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{x}(t)) \cdot \mathbf{x}'(t)\, dt$$

Green's Theorem:

$$\oint_{\partial D} M\, dx + N\, dy = \iint_D (N_x - M_y)\, dx\, dy$$

Alternate version of Green's Theorem:

$$\oint_{\partial D} \mathbf{F} \cdot d\mathbf{s} = \iint_D (\nabla \mathbf{x} \mathbf{F}) \cdot \mathbf{k}\, dx\, dy$$