Math 223, Fall 2012
Review Information for Test #1

1. Logistics. The test will be in class on **Friday 9/28/12**. Bring a supply of loose-leaf paper to the test. You may bring a calculator, but you will not need one.

Wednesday's class (9/26/12) will include some time for review. Bring questions.

2. Topics. The focus of the test will be material from lectures and homeworks through Friday, September 21 (essentially chapters 1–2 of Colley). Specific topics to know:

- Vector algebra and geometry: Magnitudes, dot and cross products: not just calculations, but knowing how to use them (e.g., to find equations and parametrizations of lines and planes: [§1.2] #13–21, 28–37, 41–43; [§1.3] #17–20, 32–34, 36; [§1.4] 1–12, 25, 26; [§1.5] 1–23
- Coordinate geometry in $\mathbb{R}^n$: [§1.6] 9–12
- Level curves and surfaces. E.g., visualize the graph of a surface in terms of its level curves, or vice versa: [§2.1] 14–23
- Evaluate a limit involving multiple variables, or show that it does not exist (e.g., by changing to polar coordinates): [§2.2] 7–33
- Calculate partial derivatives, derivatives of vector-valued functions, and gradients: [§2.3] 1–33
- Know how to determine continuity and differentiability, and what they look like: [§2.2] 38–46, [§2.3] 53–58
- Find and work with equations for tangent lines/planes/spaces to the graph of a function and its level curves/surfaces/hypersurfaces: [§2.3] 37–42, 48–52, [§2.6] 16–22, 26–30
- Work with composite functions, and apply the matrix form of the Chain Rule: 1–6, 19–29
- Directional derivatives: how to calculate them either using the limit definition or the gradient formula; determine and work with the direction of greatest increase of a function: [§2.6] 1–15

Obviously, doing all the problems listed above is impractical. In order to study for the test, do a few problems on each topic. If it seems easy, go on to the next one. If it seems hard, then do more problems on it!

A couple of topics will *not* be covered on the test:

- Rigorous definition of a limit in §2.2
- Newton's Method (§2.7)

3. Formulas. The following formulas will be provided to you on the test. You don't have to memorize them, but you do need to know how and when to use them and what the notation means. (If you are working on a review problems and you need a formula that is not on the list below, then that means that you need to know it.)

Projection: For all vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$,

$$\text{proj}_{\mathbf{a}}\, \mathbf{b} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}} \right) \mathbf{a}$$

Triangle inequality: For all vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$,

$$\|\mathbf{a} + \mathbf{b}\| \leqslant \|\mathbf{a}\| + \|\mathbf{b}\|$$

Conversion between rectangular and polar coordinates:

$$\begin{cases} x = r\, \cos\theta \\ y = r\, \sin\theta \end{cases} \qquad \begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = y/x \end{cases}$$

Conversion between rectangular and spherical coordinates:

$$\begin{cases} x = \rho\, \sin\phi\, \cos\theta \\ y = \rho\, \sin\phi\, \sin\theta \\ z = \rho\, \cos\phi \end{cases} \qquad \begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \tan\phi = \sqrt{x^2 + y^2}/z \\ \tan\theta = y/x \end{cases}$$

Product and Quotient Rules:

If $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}$ are differentiable at $\mathbf{a} \in \mathbb{R}^n$, then

$$D(fg)(\mathbf{a}) = g(\mathbf{a})Df(\mathbf{a}) + f(\mathbf{a})Dg(\mathbf{a}),$$

$$D(f/g)(\mathbf{a}) = \frac{g(\mathbf{a})Df(\mathbf{a}) - f(\mathbf{a})Dg(\mathbf{a})}{g(\mathbf{a})^2} \qquad \text{(provided that } g(\mathbf{a}) \neq 0\text{)}$$

Chain Rule: If $\mathbf{F} : \mathbb{R}^n \to \mathbb{R}^m$ and $\mathbf{G} : \mathbb{R}^m \to \mathbb{R}^p$ are functions such that $\mathbf{F}$ is differentiable at $\mathbf{a} \in \mathbb{R}^n$, and $\mathbf{G}$ is differentiable at $\mathbf{F}(\mathbf{a}) \in \mathbb{R}^m$, then

$$D(\mathbf{G} \circ \mathbf{F})(\mathbf{a}) \;=\; \big[D\mathbf{G}(\mathbf{F}(\mathbf{a}))\big]\, \big[D\mathbf{F}(\mathbf{a})\big]$$