Math 223, Fall 2010
Review Information for Test #2

1. Logistics. The test will be in class on **Friday 11/12/10**. Wednesday's class (11/10/10) will include some time for review. Bring a supply of loose-leaf paper to the test. You may bring a calculator.

2. Topics. The focus of the test will be material from lectures and homeworks through Monday, November 8 (essentially chapter 3 and sections 6.1–6.2 of Colley). Specific topics to know:

- Parametrized curves, velocity, acceleration and speed (pp. 177–181; §3.1, problems 1–11, 15–30)
- Arclength and the arclength parameter (pp. 190–194; §3.2, problems 1–12)
- The unit tangent, normal and binormal vectors; curvature and torsion (pp. 194–204; §3.2, problems 13–18, 31–38)
- Vector fields, gradient fields, potentials, flow lines (pp. 208–213; §3.3, problems 1–13, 17–30)
- Gradient, divergence, curl, what they mean geometrically, and how to work with the ∇ operator (pp. 214–218; §3.4, problems 1–16, 21–27)
- Additional problems involving topics of chapter 3 (§3.5 — useful to test your knowledge of the basics; §3.6, problems 1–6, 13–14, 26–30, 37–40, 43, 44)
- Scalar and vector line integrals: definition, different notations, how to evaluate them, and the effects of reparametrization (pp. 363–375; §6.1, problems 1–31)
- Green's Theorem: usage and application to finding areas (pp. 381–385; §6.2, problems 1–16, 19–21, 24)
- Additional problems involving topics of chapter 6 (§6.4, problems 1–5, 7, 19, 20; §6.5, problems 1–5, 24, 35)

Obviously, doing all the problems listed above is impractical. In order to study for the test, do a few problems on each topic. If it seems easy, go on to the next one. If it seems hard, then do more problems on it!

You are also responsible for all material covered on the first midterm test (chapters 1–2 of Colley) and in Math 122 (e.g., double integrals) that is necessary for the topics above.

3. Formulas. The following formulas will be provided to you on the test. You don't have to memorize them, but you do need to know how and when to use them and what the notation means. (If you are working on a review problems and you need a formula that is not on the list below, then that means that you need to know it.)

Projection: For all vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$,

$$\text{proj}_{\mathbf{a}}\, \mathbf{b} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}}\right) \mathbf{a}$$

Triangle inequality: For all vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$,

$$\|\mathbf{a} + \mathbf{b}\| \leq \|\mathbf{a}\| + \|\mathbf{b}\|$$

Conversion between rectangular and spherical coordinates:

$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases} \qquad \begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \tan\phi = \sqrt{x^2 + y^2}/z \\ \tan\theta = y/x \end{cases}$$

Product and Quotient Rules:

If $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}$ are differentiable at $\mathbf{a} \in \mathbb{R}^n$, then

$$D(fg)(\mathbf{a}) = g(\mathbf{a})Df(\mathbf{a}) + f(\mathbf{a})Dg(\mathbf{a}),$$

$$D(f/g)(\mathbf{a}) = \frac{g(\mathbf{a})Df(\mathbf{a}) - f(\mathbf{a})Dg(\mathbf{a})}{g(\mathbf{a})^2} \qquad \text{(provided that } g(\mathbf{a}) \neq 0\text{)}$$

Chain Rule: If $g : \mathbb{R}^n \to \mathbb{R}^m$ and $f : \mathbb{R}^m \to \mathbb{R}^k$ are functions such that g is differentiable at $\mathbf{a} \in \mathbb{R}^n$, and f is differentiable at $g(\mathbf{a}) \in \mathbb{R}^m$, then

$$D(f \circ g)(\mathbf{a}) = \big[Df(g(\mathbf{a}))\big]\, \big[Dg(\mathbf{a})\big]$$

Curvature:

$$\kappa = \left\|\frac{d\mathbf{T}}{ds}\right\| = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|^3}$$

Torsion:

$$\frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}; \qquad \tau = \frac{(\mathbf{v} \times \mathbf{a}) \cdot \mathbf{a}'}{\|\mathbf{v} \times \mathbf{a}\|^2}$$

Line integrals:

$$\int_{\mathbf{x}} f\, ds = \int_a^b f(\mathbf{x}(t))\, \|\mathbf{x}(t)\|\, dt \qquad \int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{x}(t)) \cdot \mathbf{x}(t)\, dt$$

Green's Theorem:

$$\oint_{\partial D} M\, dx + N\, dy = \iint_D (N_x - M_y)\, dx\, dy$$