Math 223, Fall 2010
Review Information for Final Exam

1. General Information

Time, date and place of final exam: **Monday, December 13, 10:30 AM – 1:00 PM, Wescoe 4051** (the usual classroom). Plan to arrive 15 minutes early so that we can start on time.

Bring pens or pencils, scratch paper, and a calculator. Check the batteries beforehand — two students may not share a calculator, and I will not have extras available.

The exam is closed-book and closed-notes; however, a list of formulas will be provided to you (see next page).

You don't need to bring a bluebook.

You are responsible for the following topics in addition to those covered on the two in-class tests:

- Conservative vector fields, path-independence, and the criterion involving curl (§6.3, pp. 390–396)
- Finding scalar potential functions (§6.3, pp. 396–397)
- General theory of parametrized surfaces: coordinate curves, tangent and normal vectors, tangent planes, and surface area (§7.1, pp. 405–417)
- Specific surfaces you should know how to parametrize: plane, disk, sphere, cylinder, cone (throughout chapter 7)
- Scalar and vector surface over orientable surfaces (§7.2, p. 419–435)
- Stokes' Theorem and Gauss' Theorem (§7.3, p.439–444); you don't have to know the proofs, but should understand the interpretations discussed in class

I will hold my usual office hours on Thursday 12/9 (1:00–2:15) as well as extra office hours on Friday 12/10: 10:00–12:00 and 1:00–3:00. I will also try to check my e-mail regularly over the weekend (`jmartin@math.ku.edu`).

2. Formulas that will be given to you on the exam

Projection: For all vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$,

$$\operatorname{proj}_{\mathbf{a}} \mathbf{b} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}}\right) \mathbf{a}$$

Triangle inequality: For all vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$,

$$\|\mathbf{a} + \mathbf{b}\| \leqslant \|\mathbf{a}\| + \|\mathbf{b}\|$$

Conversion between rectangular and spherical coordinates:

$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases} \qquad \begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \tan\phi = \sqrt{x^2 + y^2}/z \\ \tan\theta = y/x \end{cases}$$

Product and Quotient Rules: If $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}$ are differentiable at $\mathbf{a} \in \mathbb{R}^n$, then

$$D(fg)(\mathbf{a}) = g(\mathbf{a})Df(\mathbf{a}) + f(\mathbf{a})Dg(\mathbf{a})$$

$$D(f/g)(\mathbf{a}) = \frac{g(\mathbf{a})Df(\mathbf{a}) - f(\mathbf{a})Dg(\mathbf{a})}{g(\mathbf{a})^2} \qquad \text{(provided that } g(\mathbf{a}) \neq 0\text{)}$$

Chain Rule: If $g : \mathbb{R}^n \to \mathbb{R}^m$ and $f : \mathbb{R}^m \to \mathbb{R}^k$ are functions such that g is differentiable at $\mathbf{a} \in \mathbb{R}^n$, and f is differentiable at $g(\mathbf{a}) \in \mathbb{R}^m$, then

$$D(f \circ g)(\mathbf{a}) \;=\; \big[Df(g(\mathbf{a}))\big]\,\big[Dg(\mathbf{a})\big]$$

Curvature:

$$\kappa = \left\|\frac{d\mathbf{T}}{ds}\right\| = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|^3}$$

Torsion:

$$\frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}; \qquad \tau = \frac{(\mathbf{v} \times \mathbf{a}) \cdot \mathbf{a}'}{\|\mathbf{v} \times \mathbf{a}\|^2}$$

Line integrals:

$$\int_{\mathbf{x}} f\, ds = \int_a^b f(\mathbf{x}(t))\, \|\mathbf{x}(t)\|\, dt \qquad \int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{x}(t)) \cdot \mathbf{x}(t)\, dt$$

Green's Theorem:

$$\oint_{\partial D} M\, dx + N\, dy = \iint_D (N_x - M_y)\, dx\, dy$$

Surface integrals:

$$\iint_{\mathbf{X}} f\, dS = \iint_D f(\mathbf{X}(s,t))\, \|\mathbf{N}(s,t)\|\, ds\, dt \qquad \iint_{\mathbf{X}} \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F}(\mathbf{X}(s,t)) \cdot \mathbf{N}(s,t)\, ds\, dt$$

Stokes' Theorem:

$$\iint\limits_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \oint\limits_{\partial S} \mathbf{F} \cdot d\mathbf{s}$$

Divergence Theorem:

$$\iiint\limits_B (\nabla \cdot \mathbf{F}) \cdot dV = \oiint\limits_{\partial B} \mathbf{F} \cdot d\mathbf{S}$$

Two useful integrals (I may supply other integral formulas as needed):

$$\int \sin^2 u \, du = \frac{u - \sin u \cos u}{2} + C \qquad \int \cos^2 u \, du = \frac{u + \sin u \cos u}{2} + C$$

3. Review Exercises

For each topic, I've listed some exercises from the textbook that might resemble exam problems. Some of these exercises have already appeared in homework assignments. **The problems on the final exam will not necessarily look like these review exercises!** You should not necessarily do all the problems listed (which you won't have time for anyway). One strategy is to pick one topic at a time and work on the review exercises until they start to become boring (which is a sign that you are more familiar with the topic in question).

Chapter 1:

- Find a parametric description of a line in $\mathbb{R}^3$, given two points on it, or a point and a direction vector, or two planes that both contain the line, etc. (§1.2: #13–20)
- Find the distance between a point and a line, or between two skew lines in $\mathbb{R}^3$, or between a point and a plane in $\mathbb{R}^3$, etc. (§1.5: #20–28)
- Prove simple vector identities involving dot and/or cross products, projections, and norms: (§1.3: #17–19; §1.4: #20; §1.6: #9–12)

Chapter 2:

- Sketch the graph and/or level curves of a function $f : X \subseteq \mathbb{R}^2 \to \mathbb{R}$ (§2.1: #10–19), or the level surfaces of a function $f : X \subseteq \mathbb{R}^3 \to \mathbb{R}$ (§2.1: #28–32)
- Evaluate a limit of a multivariable function, or prove that it does not exist (§2.2: #7–22, 28–33)
- Understand what it means for a function to be continuous at a point in its domain (§2.2: #34–42)
- Calculate the derivative matrix of a function (§2.3: #20–25)
- Find the tangent plane to the graph of a function $f : X \subseteq \mathbb{R}^2 \to \mathbb{R}$ (§2.2: #29–33) or to a level surface of a function $f : X \subseteq \mathbb{R}^3 \to \mathbb{R}$ (§2.6: #16–23)
- Understand what it means for a function to be of class C^k, and what that implies about its partial derivatives (§2.4: #9–18)
- Apply the Chain Rule in its matrix form (§2.5: #15–21)
- Calculate directional derivatives and the gradient, and determine the direction of greatest increase or decrease of a function (§2.6: #2–15)

Chapter 3:

- Sketch a parametrized curve in $\mathbb{R}^2$ or $\mathbb{R}^3$ (§3.1: #1–6)
- Determine the tangent vector (or tangent line), acceleration vector, and speed of a parametrized curve at a point (§3.1: #7–10, 15–19)
- Set up an integral for the arclength of a parametrized curve, and evaluate it if possible (§3.2: #1–10)
- Sketch a vector field in $\mathbb{R}^2$ or $\mathbb{R}^3$ (§3.3: #1–12)
- Given a vector field $\mathbf{F}$, verify that a given parametrized curve is a flow line of $\mathbf{F}$ (§3.3: #17–19) or set up (and if possible solve) a system of differential equations to find flow lines of $\mathbf{F}$ (§3.3: #20–22)
- Calculate the divergence and curl of a vector field (§3.4: #1–11)
- Prove identities involving divergence and curl (§3.4: Theorem 4.3 and Theorem 4.4 (p. 218), and #14,15,21–24)

Chapter 6:

- Set up and evaluate a scalar or vector line integral of a function $f : \mathbb{R}^2 \to \mathbb{R}$ along a parametrized curve in $\mathbb{R}^2$ (§6.1: #1–11, 13, 14, 17–21)
- Understand what happens to a line integral when the curve is reparametrized (in particular, know what is meant by an orientation of a curve) (§6.1: #15)
- Use Green's Theorem to convert a line integral to a double integral, or vice versa (§6.2: #5–8,11)
- Use Green's Theorem to find the area of a bounded region (§6.2: #9,10,12–16)
- Use Green's Theorem to deduce facts about line integrals of conservative vector fields (§6.2: #19–21, §6.3: #1,2)
- Determine whether a vector field is conservative, and if so find a scalar potential function (§6.3: #3–16)
- Use the scalar potential function of a conservative vector field to evaluate line integrals (§6.3: #17–24)

Chapter 7:

- For a given parametrized surface $\mathbf{X} : \mathbb{R}^2 \to \mathbb{R}^3$, find coordinate curves, tangent vectors, normal vectors and tangent planes, and express the underlying surface as the graph of a function $f : \mathbb{R}^2 \to \mathbb{R}$ or as a level surface of a function $f : \mathbb{R}^3 \to \mathbb{R}$ (§7.1: #1–4,6–11,14–17; §7.6: #1–8)
- Find the surface area of a parametrized surface (§7.1: #18–21)
- Set up and evaluate a scalar or vector surface integral along a parametrized surface in $\mathbb{R}^3$ (§7.2: #1–3,5,6,9–23; §7.6: #9–12)
- Know what happens to a surface integral when the surface is reparametrized (§7.2: #4)
- Verify Stokes' Theorem for a given surface and vector field (§7.3: #1–4)
- Verify the Divergence Theorem for a given closed surface and vector field (§7.3: #6–9
- Use Stokes' Theorem or the Divergence Theorem to replace a difficult integral with easier ones (§7.3: #5,11,12,14,16)
- Other applications of Stokes' Theorem and the Divergence Theorem (§7.6: #21,22,26,28,38)
- Understand the geometric interpretations of Stokes' Theorem and the Divergence Theorem (§7.3: #28)