Math 141 Honors Problems #14

Due date: Tuesday, 12/8/09

HP26 [4 points]

Consider the improper integrals

$$I = \int_0^1 x^{-1}\,dx, \qquad J = \int_0^1 x^{-1/2}\,dx.$$

Of course, these integrals can be evaluated (or shown to diverge) directly.

(a) What happens when you try to approximate them using the Midpoint Rule? To get a sense of what happens, you should calculate the approximations for several different values of n. (A calculator or computer will be helpful here.)

(b) Can you make an intelligent guess about the behavior of the improper integrals

$$K = \int_0^1 \frac{sin\,x}{x}\,dx, \qquad L = \int_0^1 \frac{dx}{\ln x}$$

(which cannot be evaluated by hand)?

HP27 [2 points]

Problem #29 from section 6.1 of the textbook (find the area of a lune). Note: The answer is in the back of the book on page A107, but to earn credit, you must set up and solve the problem give a correct calculation

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HP28 [3 points]

Let r and s be constants, and consider the ellipse E defined by the equation $x^2/r^2 + y^2/s^2 = 1$. (So r and s are respectively the horizontal and vertical radii of E.)

Prove that the area of E is πrs in two ways:

(1) by expressing y as a function of x and
(2) by expressing the ellipse as a parametric curve.

If it helps, you may want to start by doing a particular example, such as $r = 2$, $s = 3$, and then figuring out how to extend your answer to all possible values of r and s.

Of course, your two answers should come out the same.

In the special case $r = s$, this ellipse is a circle of radius r, so this formula generalizes the familiar formula for the area of a circle, $A = \pi r^2$.