

Math 141 Homework #3
Due Tuesday, 9/4/07
Extra Problems

#1. Suppose that $f(x) = p(x)/q(x)$ is a rational function, where $p(x)$ and $q(x)$ are polynomials.

When does $\lim_{x \rightarrow 0} f(x) = 0$? When is $\lim_{x \rightarrow 0} f(x)$ a nonzero real number? When does $\lim_{x \rightarrow 0} f(x)$ not exist?

Your answer should be fully explained, and should cover rational functions as possibilities. That is, I should be able to take *any rational function at all* and use your answer to decide the value of its limit as $x \rightarrow 0$. (So, for example, it is not sufficient to only give an example of each of the three cases.)

#2. On Tuesday 8/28 in class, we examined a function $s(x)$ that swaps the first two digits of the decimal expansion of x . What about the function $r(x)$ that swaps the first and *third* digits of x (so, e.g., $r(1.23456) = 1.43256$, $r(2.121212) = 2.121212$)? For which real values of a is r continuous at a ?

Bonus problem: Let $T(x)$ be the function with domain $(0, \infty)$ defined as follows:

- $T(a/b) = 1/b$, if a/b is a fraction in lowest terms;
- $T(x) = 0$, if x is an irrational number.

So, e.g., $T(1/2) = 1/2$, $T(0.375) = 1/8$ (because $0.375 = 3/8$), $T(\pi) = 0$.

For which values of a is $T(x)$ continuous at a ?