Name:_____ **Instructor:**_____

Math 121 - Sample Final Exam - F07

Problem	1-15	16	17	18	19	20	21	22	23	24	Total
Points	75	40	40	40	15	20	20	25	15	10	
Score											

Instructions: You will be given 150 minutes for this exam. Part A has 15 questions, Part B has 3 problems, and Part C has 6 problems. The problems are worth a total of 300 points, with each problem weighted as indicated. Your test should have 10 pages. Write your name at the top of each page. Do not detach this sheet from the test booklet. Do all your work in this booklet and circle your final answers and write them in the appropriate boxes.

In Part A, no partial credit will be assigned. You do not need to justify your work in this part of the test. Select the most appropriate answer. Keep in mind that if you use your calculator you may only get an approximate value, while the choices given in some problems may list the exact value.

In Part B, all the answers need to be exact and complete. You do not need to justify your work in this part of the test. Partial credit might be given. **Always give the exact answers** for the problems in Part B.

In Part C, all the answers need to be exact. Partial credit might be given, and answers without sufficient justification might receive no credit. **Always give the exact answers and justify your work** for the problems in Part C.

You are allowed to use a calculator without symbolic algebra capabilities for all parts of the exam. Make sure that your calculator has been approved by your instructor.

During the exam, you are not allowed to borrow or interchange calculators. You are not allowed to use any books and notes. Turn off your cell phone and any other electronic devices (but a calculator) and keep them in your pocket or bag.

Part A: Multiple Choice Answers. After you are done with Part A **you must** copy the letters corresponding to your answers in the space below. Your answers in the space below will be used in grading Part A of the exam; so make sure you copy them correctly.

1. _____ **2.** _____ **3.** _____ **4.** _____ **5.** _____

6. _____ **7.** _____ **8.** _____ **9.** _____ **10.** _____

11. _____ **12.** _____ **13.** _____ **14.** _____ **15.** _____

Part A Total _____

Part A - Multiple Choice Examination

Select only **one** *answer for each problem. Copy your answers to the cover page!*

1. Let

$$f(x) = \begin{cases} x + c & \text{if } x < 0 \\ e^x & \text{if } x \geq 0. \end{cases}$$

The function $f(x)$ is continuous at $x = 0$ if $c =$

(A) 0 (B) 1 (C) 2 (D) -1 (E) e
(F) None of the choices (A) - (E) is correct.

2. Suppose that $f(x)$ is differentiable at $x = 2$ and satisfies $f(2) = 1$ and $f'(2) = -2$. Then the linearization of $f(x)$ at $x = 2$ is

(A) $L(x) = -3 - 2x$ (B) $L(x) = 4 - 2x$ (C) $L(x) = -4 + x$ (D) $L(x) = 3 + x$
(E) $L(x) = 5 - 2x$ (F) None of the choices (A) - (E) is correct.

3. The function $f(t)$ is continuous on the interval $[a, b]$ and satisfies $f(a) > 0$, $f(b) < 0$. Which one of the following statements MUST be true?

(A) $\int_a^b f(t)\,dt$ does not exist.
(B) $\int_a^b f(t)\,dt = 0$.
(C) $f'(t) < 0$ for every t in the interval (a, b).
(D) $f(t)$ is concave upward on the interval (a, b).
(E) $f(c) = 0$ for some number c in the interval (a, b).
(F) None of the choices (A) - (E) is correct.

4. Suppose that $f(x)$ is differentiable on the interval $[-1, 2]$, and $f(-1) = 0$, $f(2) = 0$. Which one of the following statements MUST be true?

(A) $\int_{-1}^2 f(x)\,dx = 0$.
(B) $f'(x) = 0$ for every x in the interval $(-1, 2)$.
(C) $f'(c) = 0$ for some number c in the interval $(-1, 2)$.
(D) $f(x) = 0$ for every x in the interval $(-1, 2)$.
(E) $f(c) = 0$ for some number c in the interval $(-1, 2)$.
(F) None of the choices (A) - (E) is correct.

5. The limit $\lim_{x \to 0} \dfrac{e^x - \cos x}{x} =$

(A) -1 (B) 1 (C) $-\infty$ (D) ∞ (E) Does not exist (F) None of the choices (A) - (E) is correct.

6. The limit $\lim_{x\to\infty} \dfrac{2x^2 - \sqrt{x}}{(x+\pi)^2} =$

(A) 0 (B) 1 (C) 2 (D) ∞ (E) $-\infty$ (F) None of the choices (A) - (E) is correct.

7. According to the children's growth chart, the heights of two-year-old boys is normally distributed with a mean of 34.5 inches and a standard deviation of 1.3 inches. If a two-year-old boy is selected at random, what is the probability that his height will be greater than 36.5 inches tall?

(A) .06 (B) .04 (C) .1 (D) .02 (E) .08

8. Using Newton's method to compute the root of the equation $x - 3\ln x = 0$ with initial approximation $x_1 = 1$, which of the following is nearest the third approximation x_3:

(A) 0 (B) 3 (C) 1.857 (D) 1.552 (E) 1.783 (F) None of these.

9. Air is being pumped into a spherical balloon. The radius of the balloon is increasing at a rate of 3 ft/sec. How fast is the volume of the balloon increasing with respect to time when $r = 2$ feet?

(A) 16π ft^3/s (B) $\dfrac{16\pi}{3}$ ft^3/s (C) 48π ft^3/s (D) 0 ft^3/s (E) 32π ft^3/s
(F) None of the choices (A) - (E) is correct.

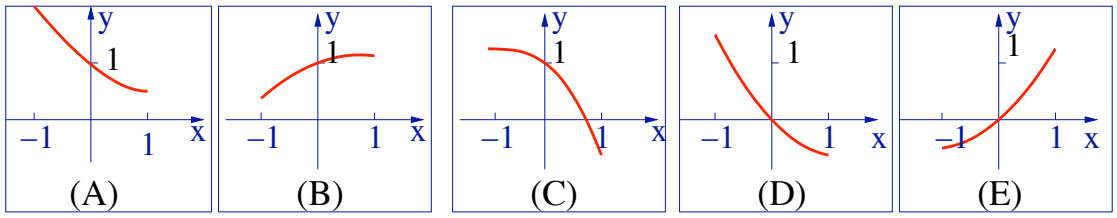
10. The cost (in dollars) for producing x units of a certain product is $C(x) = 10000 + 25x^2$. The production level that minimizes the average cost is $x =$

(A) 400 (B) 20 (C) 0 (D) 40 (E) 4000 (F) None of these

11. The absolute minimum value of the function $f(x) = e^{(2x-x^2)}$ on interval $[0, 2]$ is

(A) 1 (B) e (C) 2 (D) e^2 (E) e^{-2} (F) None of the choices (A) - (E) is correct.

12. Suppose that $f(x)$ is differentiable. If $f(0) = 1$, $f'(0) > 0$, and $f'(x)$ is decreasing on the interval $(-1, 1)$, circle one graph that best represents the curve of $f(x)$.



(F) None of the choices (A) - (E) is correct.

13. A particle is moved along the x-axis by a force that measures $f(x) = \cos x$ pounds at a point x feet from the origin. Then the work done in moving the particle from $x = 0$ to $x = \pi/2$ feet is

(A) 0 lb-ft (B) 1 lb-ft (C) 1.5 lb-ft (D) 2 lb-ft (E) 2.5 lb-ft
(F) None of the choices (A) - (E) is correct.

14. Suppose that $f(x)$ is continuous on the interval $[0, 2]$. If $\int_0^1 f(x)\,dx = 5$ and $\int_1^2 f(x)\,dx = -6$, then $\int_0^2 (2x + f(x))\,dx =$

(A) -3 (B) -1 (C) 0 (D) 1 (E) 3 (F) None of the choices (A) - (E) is correct.

15. The length of the parametric curve given by $x = 2\cos t$, $y = \sin t$ with $0 \le t \le 1$ is

(A) $\int_0^1 (\cos t - 2\sin t)\,dt$ (B) $\int_0^1 \sqrt{3 + \cos^2 t}\,dt$ (C) $\int_0^1 \sqrt{3 + \sin^2 t}\,dt$
(D) $\int_0^1 \sqrt{1 + 3\sin^2 t}\,dt$ (E) $\int_0^1 \sqrt{1 + 3\cos^2 t}\,dt$ (F) None of the choices (A) - (E) is correct.

Part B -Fill in the blanks.

16. Compute the indicated derivatives.

(A) The table shows several values of f and f'. Let $g(x) = f(e^x)$. Evaluate $g'(0)$.

x	$f(x)$	$f'(x)$
-1	2	4
0	-3	2
1	5	-2

Answer: $g'(0) =$

(B) Let $h(x) = [f(x)]^2$. Use the same data for f and f' given in Part (A) to evaluate $h'(-1)$.

Answer: $h'(-1) =$

(C) Consider the equation $e^y + x^2 y = 0$. Find $\dfrac{dy}{dx}$ in terms of x and y.

Answer: $\dfrac{dy}{dx} =$

(D) Let $k(x) = \displaystyle\int_5^{\sqrt{x}} \cos(t^2 + 1)\, dt$. Find $k'(x)$.

Answer: $k'(x) =$

17. Compute the indicated integrals.

(A) $\int xe^{(1-x^2)}\,dx.$

Answer:

(B) $\int (x-1)(\sin x)\,dx.$

Answer:

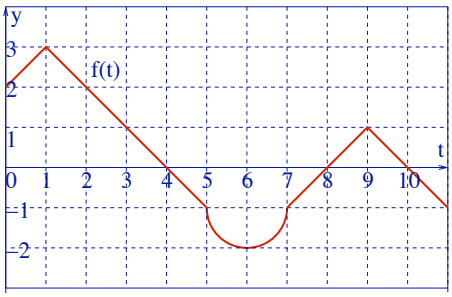
(C) $\int_1^2 \dfrac{x+1}{x^2}\,dx$ (**Give the exact value.**)

Answer:

(D) $\int_0^\infty \dfrac{e^x}{(e^x+1)^3}\,dx$ (**Give the exact value if it is convergent, or show it is divergent.**)

Answer:

18. Let $F(x) = \int_0^x f(t)\,dt$, where f is the function consists of straight lines and a semicircle shown in the following figure. **For each question write your answer in the box, and explain in the blank space.**



(A) Determine the values of $f(9)$, $F(9)$, and $F'(9)$.

Answer: $f(9) =$ $F(9) =$ $F'(9) =$

(B) List all the x value(s) at which $F(x)$ attains a local maximum value.

Answer: $x =$

(C) List all the interval (s) on which $F(x)$ is decreasing.

Answer:

(D) List all the interval(s) on which $F(x)$ is concave upward.

Answer:

Part C – Essay Questions - SHOW YOUR WORK!!

19. In three hours, the velocity of a car at each half hour was recorded as follows:

Time (hours)	0	.5	1	1.5	2	2.5	3
Velocity (mi/h)	25	45	50	55	40	45	37

Estimate of the **average** velocity (in miles/hour) of the car over these three hours using Simpson's Rule for $n = 6$.

Average velocity =

20. A cylindrical can is to be made with top and bottom made of copper costing 2 cents per square inch and curved side made of aluminum costing 1 cent per square inch. The total cost is to be no more than 300π cents total. Find the dimensions of the can that will maximize its volume.

height = radius =

21. A car was traveling along a straight road when the driver applied the brakes (at $t = 0$). The velocity of the car t seconds after the brakes were applied was

$$v(t) = 200 - 50t^2 \quad \text{ft/s}.$$

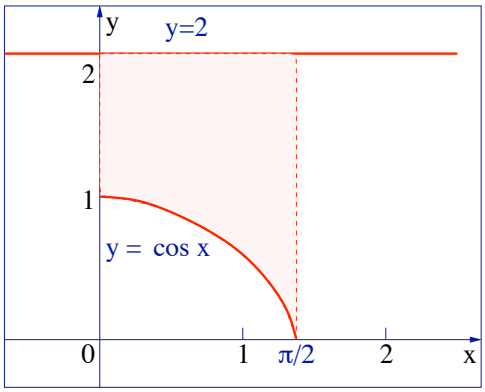
(A) What was the acceleration of the car one second after the brakes were applied?

(B) How far did the car skid from the time the brakes were applied until it stopped?

22. A tank in the shape of a circular cone with its vertex pointing downward (and its top horizontal) is completely filled with water. Assume that the radius of its circular top is 1 m and its height (i.e. the distance from the vertex to the top) is 2 m.

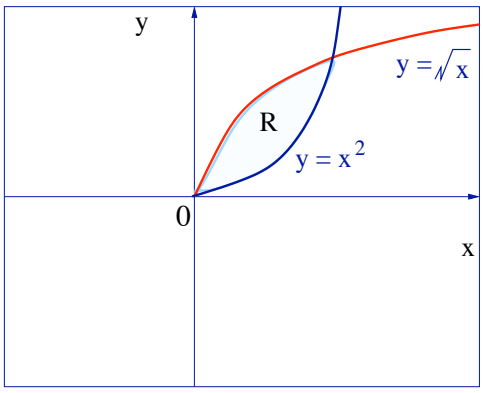
What is the work, measured in newton-meter or joules needed to pump all of the water out of this tank over its top? (Note that the density of water is 1000 kg/m^3 and the gravitational acceleration is 9.8 m/s^2.)

23. Find the area of the region bounded by the curve $y = \cos x$ and the lines $y = 2$, $x = 0$, and $x = \dfrac{\pi}{2}$.



Area =

24. Let R be the region enclosed by the curves $y = \sqrt{x}$ and $y = x^2$. Find the volume of the solid obtained by rotating R about the x-axis.



Volume =