The Divider-Chooser Method

Suppose that two players want to divide a set S of goods fairly.

One player is the **divider** (D) and one is the **chooser** (C). (Flip a coin to determine who is who.)

Step 1: D divides the booty S into two shares.

Step 2: C chooses one of the two shares for him/herself. D gets the other share.

- This is the "classic" fair-division method
- Applies to **two-player**, **continuous** fair-division games.

The Divider-Chooser Method

- Player P_1 (Divider) can guarantee himself a fair share by making sure the shares are of equal value in his opinion, so that either one will be a fair share.

- Player P_2 (Chooser) can guarantee herself a fair share by picking whichever she likes better, so that it is worth at least half the value of S in her opinion.

- Therefore, the Divider-Chooser method is **guaranteed** to yield a fair division, **regardless of the players' value systems**.

The Divider-Chooser Method: Notes

- In case you're wondering: The Divider-Chooser Method actually still works even without the privacy assumption.

- Slight drawback: The method is asymmetrical — it's typically better to be Chooser than Divider. How might we fix this?

The Divider-Chooser Method: Notes

- In case you're wondering: The Divider-Chooser Method actually still works even without the privacy assumption.

- Slight drawback: The method is asymmetrical — it's typically better to be Chooser than Divider. How might we fix this?

Big Question: What if there are more than two players?

Multiple Players: The Lone-Divider Method

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The **Lone-Divider Method** is a fair-division method that works for **multiple-player, continuous** fair-division games.

The Lone-Divider Method: Example 1

Example: Helga, Igor and Jade are trying to divide a cake fairly. They draw straws and Helga ends up as the *divider*. Igor and Jade are the choosers.

Step 1: Division. Helga cuts the cake into three shares s_1, s_2, s_3 that she considers of equal value.

The Lone-Divider Method: Example 1

Step 2: Bidding. First, each player decides (privately) on his or her valuation of each share.

		Shares		
		s_1	s_2	s_3
	Helga	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$
Players	Igor	20%	40%	40%
	Jade	40%	30%	30%

- Each row has to add up to 100% (by the Rationality Assumption).

The Lone-Divider Method: Example 1

Step 2: Bidding. The players then bid by declaring which shares they consider to be fair.

	s_1	s_2	s_3	**Bid**
Helga	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	s_1, s_2, s_3
Igor	20%	40%	40%	s_2, s_3
Jade	40%	30%	30%	s_1

The Lone-Divider Method: Example 1

Step 3: Distribution. In this case, it is possible to allocate everyone a fair share. In fact, there are two possibilities.

The Lone-Divider Method: Example 1

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Possibility 1: Helga gets s_2, Ivan gets s_3, Jade gets s_1.

	s_1	s_2	s_3
Helga	$33\frac{1}{3}\%$	**$33\frac{1}{3}\%$**	$33\frac{1}{3}\%$
Igor	20%	40%	**40%**
Jade	**40%**	30%	30%

The Lone-Divider Method: Example 1

Step 3: Distribution. In this case, it is possible to allocate everyone a fair share. In fact, there are two possibilities.

Possibility 2: Helga gets s_3, Ivan gets s_2, Jade gets s_1.

	s_1	s_2	s_3
Helga	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	**$33\frac{1}{3}\%$**
Igor	20%	**40%**	40%
Jade	**40%**	30%	30%

The Lone-Divider Method: Example 1

Step 3: Distribution. In this case, it is possible to allocate everyone a fair share. In fact, there are two possibilities.

Possibility 2: Helga gets s_3, Ivan gets s_2, Jade gets s_1.

	s_1	s_2	s_3
Helga	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	**$33\frac{1}{3}\%$**
Igor	20%	**40%**	40%
Jade	**40%**	30%	30%

What question am I going to ask next? ★

The Lone-Divider Method: Example 1

Step 3: Distribution. In this case, it is possible to allocate everyone a fair share. In fact, there are two possibilities.

Possibility 2: Helga gets s_3, Ivan gets s_2, Jade gets s_1.

	s_1	s_2	s_3
Helga	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	**$33\frac{1}{3}\%$**
Igor	20%	**40%**	40%
Jade	**40%**	30%	30%

What if Step 3 is impossible?

The Lone-Divider Method: Example 2

After Helga divides the cake (Step 1), the players' bidding (Step 2) might be as follows:

	s_1	s_2	s_3	Bid
Helga	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	s_1, s_2, s_3
Igor	40%	30%	30%	s_1
Jade	50%	25%	25%	s_1

What do we do now?

The Lone-Divider Method: Example 2

First, give Helga a share no one else wants, such as s_3.
(It would also work to give her s_2.)

Now, here comes the clever part:

The Lone-Divider Method: Example 2

First, give Helga a share no one else wants, such as s_3.
(It would also work to give her s_2.)

Now, here comes the clever part:

Recombine s_1 and s_2 into a big piece, which we'll call *b*.

Why Does This Work?

	s_1	s_2	s_3	Bid
	$\overbrace{\qquad\qquad}^{b}$ (spans s_1, s_2)			
Helga	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$s_1,\ s_2,\ s_3$
Igor	40%	30%	30%	s_1
Jade	50%	25%	25%	s_1

Why Does This Work?

- Piece b is worth 70% (40% + 30%) of the cake to Igor, and is worth 75% (50% + 25%) of the cake to Jade.

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- If Igor and Jade divide b fairly, then:

Why Does This Work?

- Piece b is worth 70% (40% + 30%) of the cake to Igor, and is worth 75% (50% + 25%) of the cake to Jade.
- If Igor and Jade divide b fairly, then:
  - Igor's piece is worth at least $\frac{1}{2} \times 70\% =$ **35%** to him
  - Jade's piece is worth at least $\frac{1}{2} \times 75\% =$ **$37\frac{1}{2}\%$** to her
- Therefore, both players will receive fair shares.

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- How should Igor and Jade divide b?

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- If Igor and Jade divide b fairly, then:
  - Igor's piece is worth at least $\frac{1}{2} \times 70\% = \mathbf{35\%}$ to him
  - Jade's piece is worth at least $\frac{1}{2} \times 75\% = \mathbf{37\frac{1}{2}\%}$ to her
- Therefore, both players will receive fair shares.
- How should Igor and Jade divide b?
  By the Divider-Chooser Method, of course!

Recap: The Lone-Divider Method

Three players: a divider (D) and two choosers (C_1 and C_2).

Step 1: Division. D divides the booty into three shares (s_1, s_2, s_3) of equal value (to D).

Step 2: Bidding. Each player declares which pieces s/he considers to be a fair share to her.

- To the divider, **any** of s_1, s_2, s_3 is a fair share.
- To each chooser, **at least one** of s_1, s_2, s_3 is a fair share.

Recap: The Lone-Divider Method

Step 3: Distribution.

If possible ("Case 1"), allocate a piece to each player so that each player receives a fair share.

If that is impossible ("Case 2"), the reason **must** be that the two choosers want the same piece, and each of them wants only that piece. That is,

- There is only one piece (the "C-piece") that both C_1 and C_2 want.
- There are two other pieces (the "U-pieces") that neither of them want.

In Case 2. . .

Recap: The Lone-Divider Method

Case 2: There is only one piece (the "C-piece") that both C_1 and C_2 consider a fair share and two other pieces (the "U-pieces") that neither of them consider a fair share.

Then, proceed as follows:

- Give D one of the U-pieces (it doesn't matter which one).
- Combine the C-piece and the remaining U-piece into a big piece (the "B-piece").
- Have C_1 and C_2 split the B-piece using the divider-chooser method.

The Lone-Divider Method: Example 3

Let's make Igor be the divider this time.
The bidding table might look like this:

	s_1	s_2	s_3
Helga	$1/2$	$1/3$	$1/6$
Igor	$1/3$	$1/3$	$1/3$
Jade	$1/2$	$1/4$	$1/4$

The Lone-Divider Method: Example 3

Let's make Igor be the divider this time.
The bidding table might look like this:

	s_1	s_2	s_3	**Bids**
Helga	**1/2**	**1/3**	1/6	s_1, s_2
Igor	**1/3**	**1/3**	**1/3**	s_1, s_2, s_3
Jade	1/4	**1/2**	1/4	s_2

The Lone-Divider Method: Example 3

Let's make Igor be the divider this time.
The bidding table might look like this:

	s_1	s_2	s_3	**Bids**
Helga	**1/2**	**1/3**	1/6	s_1, s_2
Igor	**1/3**	**1/3**	**1/3**	s_1, s_2, s_3
Jade	1/4	**1/2**	1/4	s_2

Jade gets s_1, Igor gets s_3, and Helga gets s_2.

Everyone gets a fair share!

The Lone-Divider Method: Example 4

What if we change Helga's valuation?

	s_1	s_2	s_3
Helga	$1/2$	$1/4$	$1/4$
Igor	$1/3$	$1/3$	$1/3$
Jade	$1/2$	$1/4$	$1/4$

The Lone-Divider Method: Example 4

What if we change Helga's valuation?

	s_1	s_2	s_3	**Bids**
Helga	1/4	**1/2**	1/4	s_2
Igor	**1/3**	**1/3**	**1/3**	s_1, s_2, s_3
Jade	1/4	**1/2**	1/4	s_2

The Lone-Divider Method: Example 4

What if we change Helga's valuation?

	s_1	s_2	s_3	**Bids**
Helga	1/4	**1/2**	1/4	s_2
Igor	**1/3**	**1/3**	**1/3**	s_1, s_2, s_3
Jade	1/4	**1/2**	1/4	s_2

- s_2 is the C-piece and s_1 and s_3 are the U-pieces.

The Lone-Divider Method: Example 4

What if we change Helga's valuation?

	s_1	s_2	s_3	**Bids**
Helga	1/4	**1/2**	1/4	s_2
Igor	**1/3**	**1/3**	**1/3**	s_1, s_2, s_3
Jade	1/4	**1/2**	1/4	s_2

- s_2 is the C-piece and s_1 and s_3 are the U-pieces.
- Give Igor one of the U-pieces (let's say s_1), and recombine s_2 and s_3 into the B-piece.

The Lone-Divider Method: Example 4

What if we change Helga's valuation?

	s_1	s_2	s_3	**Bids**
Helga	1/4	**1/2**	1/4	s_2
Igor	**1/3**	**1/3**	**1/3**	s_1, s_2, s_3
Jade	1/4	**1/2**	1/4	s_2

- s_2 is the C-piece and s_1 and s_3 are the U-pieces.
- Give Igor one of the U-pieces (let's say s_1), and recombine s_2 and s_3 into the B-piece.
- When Helga and Jade divide the B-piece, each gets a share worth at least $(1/2 + 1/4) / 2 =$ **3/8**, which is more than 1/3.

Things You Ought To Be Wondering At This Point

1. Why does this work?

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Things You Ought To Be Wondering At This Point

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2. What if there are more than three players?
3. What if one of the players tries to cheat?
4. What if the players don't agree on the value of S?

Things You Ought To Be Wondering #1

Why does the Lone-Divider Method work?

Why the Lone-Divider Method Works

Let's go back to Example 2, with Helga as the divider.

	s_1	s_2	s_3	Bid
Helga	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	s_1, s_2, s_3
Igor	40%	30%	30%	s_1
Jade	50%	25%	25%	s_1

The first step was to give Helga s_3, which is a U-piece (we could have given her s_2 instead).

Why the Lone-Divider Method Works (Logic)

- In Igor's opinion, s_3 is worth less than $1/3$ of S.

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- In Igor's opinion, s_3 is worth less than $1/3$ of S.
- Therefore, the B-piece (the entire cake minus s_3) is worth more than $2/3$ of S.

Why the Lone-Divider Method Works (Logic)

- In Igor's opinion, s_3 is worth less than $1/3$ of S.
- Therefore, the B-piece (the entire cake minus s_3) is worth more than $2/3$ of S.
- Therefore, when Igor and Jade divide the B-piece fairly, Igor is guaranteed to receive at least $1/2$ the value of b, i.e., at least $1/2 \times 2/3 = 1/3$ of the value of S.

Why the Lone-Divider Method Works (Logic)

- In Jade's opinion, s_3 is worth less than $1/3$ of S.
- Therefore, the B-piece (the entire cake minus s_3) is worth more than $2/3$ of S.
- Therefore, when Igor and Jade divide the B-piece fairly, Jade is guaranteed to receive at least $1/2$ the value of b, i.e., at least $1/2 \times 2/3 = 1/3$ of the value of S.

(The same logic applies to Jade as well as Igor.)

Why the Lone-Divider Method Works (Algebra)

In Igor's opinion,

$$s_3 < \frac{S}{3} \ldots$$

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In Igor's opinion,

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$$\ldots \text{ so } b > \frac{2S}{3} \ldots$$

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Why the Lone-Divider Method Works (Algebra)

In Igor's opinion,

$$s_3 < \frac{S}{3} \ldots$$

$$\ldots \text{ so } S - s_3 > \frac{2S}{3} \ldots$$

$$\ldots \text{ so } b > \frac{2S}{3} \ldots$$

$$\ldots \text{ so } \frac{b}{2} > \frac{S}{3}.$$

Therefore, Igor's share will be worth **at least $\frac{1}{3}$S** to him — that is, it will be a fair share.

Things You Ought To Be Wondering #2

What if there are more than three players?

The Lone-Divider Method for 3 Players

Suppose there are 3 players.

One of the players, D, gets to be the *divider*.

The other players C_1 and C_2 are the *choosers*.

Step 1: Division. D divides the booty into N shares that he considers to be of equal value.

Step 2: Bidding. Each chooser decides (independently) which pieces she considers to be a fair share to her.

The Lone-Divider Method for 3 Players

Step 3: Distribution.

1) If possible, allocate the N pieces so that each player receives a fair share.

The Lone-Divider Method for 3 Players

Step 3: Distribution.

1) If possible, allocate the N pieces so that each player receives a fair share.

2) If that is impossible, the reason must be that C_1 and C_2 bid on the same piece (the "C-piece") and not on either of the other two pieces (the "U-pieces"). **Then:**

The Lone-Divider Method for 3 Players

Step 3: Distribution.

1) If possible, allocate the N pieces so that each player receives a fair share.

2) If that is impossible, the reason must be that C_1 and C_2 bid on the same piece (the "C-piece") and not on either of the other two pieces (the "U-pieces"). **Then:**

3a) Give the divider a U-piece.

The Lone-Divider Method for 3 Players

Step 3: Distribution.

1) If possible, allocate the N pieces so that each player receives a fair share.

2) If that is impossible, the reason must be that C_1 and C_2 bid on the same piece (the "C-piece") and not on either of the other two pieces (the "U-pieces"). **Then:**

3a) Give the divider a U-piece.

3b) Combine the other U-piece and the C-piece into a big "B-piece".

The Lone-Divider Method for 3 Players

Step 3: Distribution.

1) If possible, allocate the N pieces so that each player receives a fair share.

2) If that is impossible, the reason must be that C_1 and C_2 bid on the same piece (the "C-piece") and not on either of the other two pieces (the "U-pieces"). **Then:**

3a) Give the divider a U-piece.

3b) Combine the other U-piece and the C-piece into a big "B-piece".

3c) Then C_1 and C_2 divide the B-piece fairly.

The Lone-Divider Method for N Players

Suppose there are N players.

One of the players, D, gets to be the *divider*.

The other players C_1, C_2, ..., C_{N-2}, C_{N-1} are the *choosers*.

Step 1: Division. D divides the booty into N shares that he considers to be of equal value.

Step 2: Bidding. Each chooser decides (independently) which pieces she considers to be a fair share to her.

The Lone-Divider Method for N Players

Step 3: Distribution. This is the hard part.

- If possible, allocate the N pieces so that each player receives a fair share.
- If that is impossible, the reason must be that some number of choosers (say K of them) are fighting over $K - 1$ pieces.
- In that case, it will always be possible to give one of the non-fighters one of the pieces that aren't being fought over, reducing the fair-division problem to one with fewer players.

Four-Player Example #1

Divider Dave and Choosers Carrie, Chris, and Clara are trying to divide an avocado-liver-marshmallow pizza fairly.

Dave divides the pizza into four slices, which the players value as follows:

	s_1	s_2	s_3	s_4
Dave	25%	25%	25%	25%
Carrie	40%	20%	20%	20%
Chris	20%	30%	20%	30%
Clara	40%	10%	20%	30%

Four-Player Example #2

Divider Dave and Choosers Carrie, Chris, and Clara are trying to divide an avocado-liver-marshmallow pizza fairly.

Dave divides the pizza into four slices, which the players value as follows:

	s_1	s_2	s_3	s_4
Dave	25%	25%	25%	25%
Carrie	40%	20%	20%	20%
Chris	20%	30%	20%	30%
Clara	40%	20%	20%	20%

Things You Ought To Be Wondering #4

What if the players don't agree on the total value of S?

Things You Ought To Be Wondering #4

What if the players don't agree on the total value of S?

No problem!

Handling Differing Valuations

	s_1	s_2	s_3	s_4
Dave	\$3	\$3	\$3	\$3
Carrie	\$4	\$2	\$2	\$2
Chris	\$9	\$6	\$6	\$9
Clara	\$8	\$2	\$4	\$6

Handling Differing Valuations

	s_1	s_2	s_3	s_4
Dave	$3	$3	$3	$3
Carrie	$4	$2	$2	$2
Chris	$9	$6	$6	$9
Clara	$8	$2	$4	$6

1. Find what each player thinks the entire booty is worth.
2. Find what each share is worth as a percent of the total.
3. You will need a separate calculation for each player.

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	Monetary values					**Percentages**			
	s_1	s_2	s_3	s_4	**S**	s_1	s_2	s_3	s_4
Dave	\$3	\$3	\$3	\$3	**\$12**	**25%**	**25%**	**25%**	**25%**
Carrie	\$4	\$2	\$2	\$2	**\$10**	**40%**	**20%**	**20%**	**20%**
Chris	\$9	\$6	\$6	\$9	**\$30**	**30%**	**20%**	**20%**	**30%**
Clara	\$8	\$2	\$4	\$6	**\$20**	**40%**	**10%**	**20%**	**30%**

Things You Ought To Be Wondering #3

What if one of the players tries to cheat?

The Consequences of Cheating

Three cattle rustlers (Dillinger, Cassidy and Clyde) plan to divide a herd of stolen cows using the Lone-Divider method. Dillinger divides the herd into three shares, which the players value[1] as follows:

	s_1	s_2	s_3	**Bid**
Dillinger	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	s_1, s_2, s_3
Cassidy	50%	20%	30%	s_1
Clyde	50%	40%	10%	s_1, s_2

[1] I have changed the numbers slightly from those I used in class on 9/30/11.

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Cassidy	**50%**	20%	30%	$\mathbf{s_1}$
Clyde	50%	**40%**	10%	s_1, $\mathbf{s_2}$

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Dillinger	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$\mathbf{33\frac{1}{3}\%}$	s_1, s_2, $\mathbf{s_3}$
Cassidy	**50%**	20%	30%	$\mathbf{s_1}$
Clyde	50%	**40%**	10%	s_1, $\mathbf{s_2}$

But what if Clyde lied?

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The Consequences of Cheating

	s_1	s_2	s_3	**Bid**
Dillinger	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	s_1, s_2, s_3
Cassidy	50%	20%	30%	s_1
Clyde (Liar!)	50%	40%	10%	s_1

The Consequences of Cheating

	s_1	s_2	s_3	**Bid**
Dillinger	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	s_1, s_2, s_3
Cassidy	50%	20%	30%	s_1
Clyde (Liar!)	50%	40%	10%	s_1

- The C-piece is s_1 and the U-pieces are s_2 and s_3.

The Consequences of Cheating

	s_1	s_2	s_3	**Bid**
Dillinger	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	s_1, s_2, s_3
Cassidy	50%	20%	30%	s_1
Clyde (Liar!)	50%	40%	10%	s_1

- The C-piece is s_1 and the U-pieces are s_2 and s_3.
- Dillinger gets one of the U-pieces.

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- The C-piece is s_1 and the U-pieces are s_2 and s_3.
- Dillinger gets one of the U-pieces.
- **Whether Clyde is guaranteed a fair share depends on which U-piece Dillinger gets.**

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Possibility 1: If s_3 is chosen as the U-piece...

- The B-piece consists of s_1 and s_2 together.

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- The B-piece consists of s_1 and s_2 together.
- Cassidy values the B-piece at $50\% + 20\% = 70\%$.
- Clyde values the B-piece at $50\% + 40\% = 90\%$.

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Possibility 1: If s_3 is chosen as the U-piece...

- The B-piece consists of s_1 and s_2 together.

- Cassidy values the B-piece at $50\% + 20\% = 70\%$.
- Clyde values the B-piece at $50\% + 40\% = 90\%$.

- Both players are still guaranteed a fair share (35% and 45% respectively).
- Clyde has successfully gotten more than he is entitled to (but at least he hasn't prevented Cassidy from getting a fair share).

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Possibility 2: If s_2 is chosen as the U-piece…

- The B-piece consists of s_1 and s_3 together.

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- The B-piece consists of s_1 and s_3 together.

- Cassidy values the B-piece at $50\% + 30\% = 80\%$.
- Clyde values the B-piece at $50\% + 10\% = 60\%$.

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Possibility 2: If s_2 is chosen as the U-piece...

- The B-piece consists of s_1 and s_3 together.

- Cassidy values the B-piece at $50\% + 30\% = 80\%$.
- Clyde values the B-piece at $50\% + 10\% = 60\%$.

- Cassidy is guaranteed a fair share (40%).
- Clyde is not guaranteed a fair share:
  his eventual share may only be worth $60\% / 2 = 30\%$.

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The Punchline:

- **Bidding insincerely** can sometimes increase your share, but it can also cost you a fair share.
- **Bidding sincerely** always guarantees you a fair share (even if other players are insincere).