Ranking All The Candidates

Once we've chosen a voting method and a declared a winner, how do we know who finished second? Third? Last?

Why do this? For example:

- ▶ Electing officers: Winner becomes president, second-place finisher becomes vice-president, ...
- ▶ Draft choices: Take the highest-ranked player available

Each voting method we've studied can be used as an **extended ranking method.**

The Extended Plurality Method

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Candidate	Votes	Rank
Alesha	14	1st place
Boris	4	4th place
Carmen	11	2nd place
Dave	8	3rd place

The Extended Borda Count

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Candidate	Borda Count	Rank
Alesha	79	**4th place**
Boris	106	**1st place**
Carmen	104	**2nd place**
Dave	81	**3rd place**

The Extended Method of Pairwise Comparisons

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Candidate	Would beat	Comparisons won	Rank
Alesha	Nobody	0	**4th place**
Boris	A, D	2	**2nd place**
Carmen	A, B, D	3	**1st place**
Dave	A	1	**3rd place**

The Extended PWE Method

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Candidate	When eliminated	Rank
Alesha	Third round	**2nd place**
Boris	First round	**4th place**
Carmen	Second round	**3rd place**
Dave	Never	**1st place**

Comparing Extended Ranking Methods

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Candidate	Plurality	Borda	PComps	PWE
Alesha	**1st**	4th	4th	2nd
Boris	4th	**1st**	2nd	4th
Carmen	2nd	2nd	**1st**	3rd
Dave	3rd	3rd	3rd	**1st**

There is another approach to turn any voting method into a ranking method, using the idea of *recursion*.

We are skipping this material (pp. 24–26 of Tannenbaum); you are not responsible for it on the homework or tests.

Which Voting Method Is The Best?

Which voting method is best?

There is no purely mathematical answer to this question.

Arrow's Theorem: There is no voting method that always satisfies all four voting criteria – Majority, Condorcet, Monotonicity and IIA.

So, the answer depends which fairness criteria you think are the most important.

- ▶ Be skeptical of claims that "the vast majority of mathematical experts on voting theory believe that X is the best method."

Advocates of Various Voting Methods

We're going to look at the way that different voting methods are portrayed by their advocates.

Disclaimer: I am not responsible for the content of external websites and neither endorse or oppose any non-mathematical opinions thereon.

The Borda count

Prof. Don Saari
Prominent mathematician/physicist; advocate of Borda Count

- ▶ Fun short read: Talking about voting with fourth graders

- ▶ Sample article: "The optimal ranking is the Borda count"

Despite the title, what Saari proves that **if you are going to assign points for places,** then the Borda count is the only way that, e.g., cannot rank the Condorcet winner last.

So, this article says nothing about comparing Borda with PWE or Pairwise Comps

The Method of Pairwise Comparisons

CIVS (Condorcet Internet Voting Project)
Advocate of Method of Pairwise Comparisons

- Criticizes PWE Method (= IRV = STV):

  ". . . the Condorcet methods supported by CIVS are better than IRV at identifying consensus candidates. STV can elect a candidate even though a majority of voters would prefer someone else, and it has in real elections."

The Plurality-with-Elimination Method

Center for Voting and Democracy
Advocate of Plurality-with-Elimination Method (= IRV)

- ▶ Compares different voting methods by different fairness criteria — which criteria do they consider most/least important? (Remember, this is a matter of opinion.)

- ▶ Criticizes other voting methods, e.g.:

  Does the method require the winner to have core support? We believe a winner should be at least one voter's first choice, meaning they would receive more than 0% in current rules.

- ▶ Argues that violations of Monotonicity are unlikely

Chapter 1 Summary

Voting systems: Plurality, Borda Count, Plurality-with-Elimination, Method of Pairwise Comparisons

Fairness criteria: Majority, Condorcet, Public-Enemy, Monotonicity, Independence-of-Irrelevant-Alternatives

Arrow's Theorem: *No voting method can always satisfy all fairness criteria.*

Gibbard-Satterthwaite Theorem: *No voting method is completely immune to strategic (insincere) voting.*

Which voting system is best? This question has no single mathematical answer, but any debate needs to be informed by mathematics.