The 2000 San Diego Mayoral Election[1]

Candidate	Percentage of Votes
Ron Roberts	25.72%
Dick Murphy	15.68%
Peter Q. Davis	15.62%
Barbara Warden	15.16%
George Stevens	10.42%
Byron Wear	9.02%
All others	8.38%

- If no candidate receives a majority, then by law, a runoff election is held between the top two candidates.

---

[1] http://www.sandiego.gov/city-clerk/pdf/mayorresults.pdf

Practical problems with this system:

- ▶ Runoff elections cost time and money
- ▶ Ties (or near-ties) for second place

- ▶ Preference ballots provide a method of holding an "instant-runoff" election: the **Plurality-with-Elimination Method** (PWE).

1. **Count** the first-place votes. If some candidate receives a *majority* of the first-place votes, then that candidate is the winner.

The Plurality-with-Elimination Method (§1.4)

1. **Count** the first-place votes. If some candidate receives a *majority* of the first-place votes, then that candidate is the winner.

2. If no candidate has received a majority, then **eliminate** the candidate with the *fewest* first-place votes.

1. **Count** the first-place votes. If some candidate receives a *majority* of the first-place votes, then that candidate is the winner.

2. If no candidate has received a majority, then **eliminate** the candidate with the *fewest* first-place votes.

3. **Repeat** steps 1 and 2 until some candidate has a majority, then declare that candidate the winner.

The Elimination Assumption

Reminder: The Elimination Assumption says that when a candidate is eliminated, a voter's other relative preferences remain the same.

Ballot
1st *C*
2nd *B*
3rd *D*
4th *A*

Ballot
1st *C*
~~2nd *B*~~
3rd *D*
4th *A*

➡️

Ballot
1st *C*
2nd *D*
3rd *A*

Original ballot   Eliminating Candidate B

An Example of the PWE Method

Round 1:

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Round 1:

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

First-place votes: A: 14   B: 4   C: 11   D: 8

$\implies$ B is eliminated.

An Example of the PWE Method

Round 1:

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B̶	C
2nd choice	B̶	B̶	C	D	D
3rd choice	C	D	B̶	C	B̶
4th choice	D	A	A	A	A

First-place votes: A: 14   B: 4   C: 11   D: 8

$\implies$ B is eliminated.

Round 2:

Number of Voters	14	10	8	4	1
1st choice	A	C	D	D	C
2nd choice	C	D	C	C	D
3rd choice	D	A	A	A	A

Round 2:

Number of Voters	14	10	8	4	1
1st choice	A	C	D	D	C
2nd choice	C	D	C	C	D
3rd choice	D	A	A	A	A

First-place votes: A: 14   C: 11   D: 12

$\implies$ C is eliminated.

Round 2:

Number of Voters	14	10	8	4	1
1st choice	A	C̶	D	D	C̶
2nd choice	C̶	D	C̶	C̶	D
3rd choice	D	A	A	A	A

First-place votes: A: 14   C: 11   D: 12

$\implies$ C is eliminated.

Round 3:

Number of Voters	14	10	8	4	1
1st choice	A	D	D	D	D
2nd choice	D	A	A	A	A

Round 3:

Number of Voters	14	10	8	4	1
1st choice	A	D	D	D	D
2nd choice	D	A	A	A	A

First-place votes: A: 14   D: 23

$\implies$ **D is the winner!**

Different Methods, Different Results

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

- Plurality winner: A
- Borda count winner: B
- Condorcet winner: C
- PWE winner: D

Another PWE Example

Example: The Small Seven Athletic Conference would like to add an eighth member. The possibilities are Alaska (A), Harvard (H), North Dakota (N), and Susquehanna (S).

All students at SSAC schools are polled. The results:

Percentage of Voters	35%	30%	20%	15%
1st choice	H	S	N	S
2nd choice	N	A	A	N
3rd choice	A	H	H	A
4th choice	S	N	S	H

Who is the winner under PWE?

Does the PWE Method satisfy the Majority Criterion?

That is, if Candidate X receives a **majority** of the **first-place votes**, will X **definitely win** under the PWE Method?

Does the PWE Method satisfy the Majority Criterion?

That is, if Candidate X receives a **majority** of the **first-place votes**, will X **definitely win** under the PWE Method?

Yes. In fact, if X is a majority candidate, then X wins in the first round (i.e., before any elimination takes place).

Does PWE satisfy the Public-Enemy Criterion?

That is, if Candidate Y receives a **majority** of the **last-place votes**, is it **impossible for Y to win** under PWE? ⭐

PWE and Fairness Criteria

Does PWE satisfy the Public-Enemy Criterion?

That is, if Candidate Y receives a **majority** of the **last-place votes**, is it **impossible for Y to win** under PWE? ⭐

Yes. Even if Y survives until the last round, Y will certainly be eliminated at that stage (since whoever else survives will beat Y head-to-head.)

PWE and Fairness Criteria

Does PWE satisfy the Condorcet Criterion?

That is, if Candidate Z would beat any other candidate **head-to-head**, will X **definitely win** under the PWE Method?

PWE and Fairness Criteria

Does PWE satisfy the Condorcet Criterion?

That is, if Candidate Z would beat any other candidate **head-to-head**, will X **definitely win** under the PWE Method?

No. For example, in the Math Club election, the Condorcet candidate did not win under the PWE Method.

Reminder: This does not mean it's *impossible* for a Condorcet candidate to win under PWE! It just means that being a Condorcet candidate does not *guarantee* winning under PWE.

- ▶ PWE uses preference ballots to hold an "instant runoff" election (unlike a traditional runoff election, which takes place in multiple stages).

- ▶ PWE satisfies the Majority and Public-Enemy Criteria, but not the Condorcet Criterion.

Comparison of Voting Methods

	Fairness Criteria		
	Majority	Condorcet	Public-Enemy
Plurality	**Yes**	**No**	**No**
Borda Count	**No**	**No**	**Yes**
PWE	**Yes**	**No**	**Yes**

- ▶ Does any voting method satisfy the Condorcet Criterion?
- ▶ Are there other fairness criteria to study?

A Problem with PWE (Example 1.10)

The site of the 2116 Olympics is to be chosen using PWE.
The finalists are Athens, Barcelona, and Calgary.

A straw poll yields the following preference schedule:

Number of voters	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

If the election were held right now:

A Problem with PWE (Example 1.10)

The site of the 2116 Olympics is to be chosen using PWE.
The finalists are Athens, Barcelona, and Calgary.

A straw poll yields the following preference schedule:

Number of voters	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

If the election were held right now:

▶ Round 1: Athens 11, Barcelona 8, Calgary 10. Barcelona is eliminated.

A Problem with PWE (Example 1.10)

The site of the 2116 Olympics is to be chosen using PWE.
The finalists are Athens, Barcelona, and Calgary.

A straw poll yields the following preference schedule:

Number of voters	7	8	10	4
1st	A	~~B~~	C	A
2nd	~~B~~	C	A	C
3rd	C	A	~~B~~	~~B~~

If the election were held right now:

- Round 1: Athens 11, Barcelona 8, Calgary 10. Barcelona is eliminated.

A Problem with PWE (Example 1.10)

The site of the 2116 Olympics is to be chosen using PWE.
The finalists are Athens, Barcelona, and Calgary.

A straw poll yields the following preference schedule:

Number of voters	7	8	10	4
1st	A	B̶	C	A
2nd	B̶	C	A	C
3rd	C	A	B̶	B̶

If the election were held right now:

- ▶ Round 1: Athens 11, Barcelona 8, Calgary 10. Barcelona is eliminated.
- ▶ Round 2: Athens 11, Calgary 18. **Calgary wins**.

A Problem with PWE (Example 1.10)

The bloc of 4 voters (who are from Detroit), who had ranked Athens first and Calgary second, now decide to switch their votes.

(Clearly Calgary is going to win, so the Detroit bloc hopes that the Calgary contingent will support Detroit next time!)

Straw poll

# voters	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

Actual vote

# voters	7	8	14
1st	A	B	C
2nd	B	C	A
3rd	C	A	B

Straw poll				
# voters	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

Actual vote

# voters	7	8	14
1st	A	B	C
2nd	B	C	A
3rd	C	A	B

Straw poll				
# voters	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

Actual vote			
# voters	7	8	14
1st	A	B	C
2nd	B	C	A
3rd	C	A	B

▶ Round 1: Athens 7, Barcelona 8, Calgary 14. Athens is eliminated.

Straw poll				
# voters	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

Actual vote

# voters	7	8	14
1st	A̶	B	C
2nd	B	C	A̶
3rd	C	A̶	B

▶ Round 1: Athens 7, Barcelona 8, Calgary 14. Athens is eliminated.

Straw poll				
# voters	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

Actual vote

# voters	7	8	14
1st	~~A~~	B	C
2nd	B	C	~~A~~
3rd	C	~~A~~	B

- ▶ Round 1: Athens 7, Barcelona 8, Calgary 14. Athens is eliminated.
- ▶ Round 2: Barcelona 15, Calgary 14. **Barcelona wins**.

Straw poll				
# voters	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

Actual vote			
# voters	7	8	14
1st	~~A~~	B	C
2nd	B	C	~~A~~
3rd	C	~~A~~	B

▶ Round 1: Athens 7, Barcelona 8, Calgary 14. Athens is eliminated.

▶ Round 2: Barcelona 15, Calgary 14. **Barcelona wins**.

How could **more** votes for Calgary cause them to lose the election??

This example illustrates that the PWE Method fails the following fairness criterion:

The Monotonicity Criterion: If X is the winner of an election, and the ballots are then changed in a way that favors X and only X, then X *should* still win the election.

The Monotonicity Criterion

This example illustrates that the PWE Method fails the following fairness criterion:

The Monotonicity Criterion: If X is the winner of an election, and the ballots are then changed in a way that favors X and only X, then X *should* still win the election.

- ▶ The PWE Method **fails** the Monotonicity Criterion.
- ▶ The Plurality Method **satisfies** the Monotonicity Criterion.
- ▶ The Borda Count **satisfies** the Monotonicity Criterion.

The Monotonicity Criterion: If X is the winner of an election, and the ballots are then changed in a way that favors X and only X, then X *should* still win the election.

The Monotonicity Criterion is important because...

- ▶ A candidate should not "do worse by doing better".
- ▶ A voting method that fails the Monotonicity Criterion may be vulnerable to strategic/insincere voting.

Comparison of Voting Methods

Fairness Criteria

	Majority	Condorcet	Public-Enemy	Monotonicity
Plurality	**Yes**	**No**	**No**	**Yes**
Borda	**No**	**No**	**Yes**	**Yes**
PWE	**Yes**	**No**	**Yes**	**No**

▶ Is there a voting system that satisfies the Condorcet Criterion?